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Vol. 245 FEBRUARY, 1948 No. 2

GUIDED BOMBS IN WORLD WAR II.[1]

BY

L. O. GRONDAHL, Ph.D., Sc.D.(Hon.).*

This paper is based in whole or in part on work done for the Office of Scientific Research and Development under Contracts OEMsr-240 and OEMsr-1013 with Massachusetts Institute of Technology, under Contract NDCrc-183 with the Gulf Research and Development Corporation, and under Contracts OEMsr-1081, OEMsr-1285, and OEMsr-1415 with the Union Switch and Signal Company.

My purpose is to tell you the story of a project of the war years which was completed from the initial research stages through development, engineering, and production to effective combat service in $3\frac{1}{2}$ years. The achievement can probably be matched by many others of that time and is therefore only a sample of what can be done when it is appropriate to follow long shots and to make all the false starts—and the good ones—simultaneously. The story is interesting as an illustration both of effective cooperation of a great number of organizations and of quick results. Such a development in industry usually takes 10 to 20 years to complete. The story has to do with the development of a dirigible high angle bomb. Three varieties were completed, but only one went into actual combat service. The research work was conducted at colleges and universities, at aeronautical laboratories, air fields, industrial laboratories, and Army establishments.

The laboratories and the air fields were distributed over the country from Boston to Los Angeles, and at many points between these extremes. We shall mention some of them as we proceed with the story. The men who had to do with the promotion of the use of the bombs in

[1] This paper consists of the essential portions of a talk before the Franklin Institute on October 15, 1947. Some of this material was published in *Ordnance*, May–June, 1947, p. 503.

* Consultant in Research, Westinghouse Air Brake Company and Union Switch and Signal Company. Formerly a member of Section D-3 and Division 5, and Chief of Section 5.2, National Defense Research Committee.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

the combat zones flew to London, France, Germany, North Africa, Sicily, and Italy to the east, and to Hawaii, China, and Burma to the west.

The principal contributions to the project were made by the National Defense Research Committee, which provided the funds and the administration: Massachusetts Institute of Technology at which a small group of physicists made the early experiments, and at which much development of the fundamental apparatus, wind tunnel tests, and analysis of trajectories were made; the Army Air Forces which furnished the personnel and the airplanes, and other facilities necessary for the field tests; the Gulf Research and Development Corporation which completed the development that had been started at M.I.T. and did a vast amount of analytical work and field testing; and the Union Switch and Signal Company which did the engineering for production and the final manufacture. In addition to these there were more than 20 other organizations which contributed to the development and the manufacture of component parts.

The high angle bomb is the bomb that you have seen many times in motion pictures, and which is dropped out of a bomb bay at an instant determined by the bombardier by the use of a bomb sight into which he feeds the necessary information. The most important parts of this information are the speed of the plane and the altitude. The plane flies in a straight line towards the target and the bomb is dropped at a predetermined instant, such that if everything functions exactly correctly it falls on the target. As the bomb falls away from the plane its vertical velocity increases very rapidly in response to the action of gravity. Its horizontal velocity is approximately constant and equal to that of the plane, so that if the plane is traveling 200 m.p.h. the bomb's velocity parallel with the ground will continue to be nearly 200 m.p.h. during the whole drop. Its vertical velocity may increase to 700 or 800 m.p.h. depending upon the altitude from which it is dropped. If a bomb is dropped from 15,000 ft. or higher, the last portion of its trajectory is nearly vertical.

When a bomb is dropped in this way, small errors in manipulation of the plane or of the bomb sight, or errors in information in regard to the exact altitude of the target and the exact speed of the plane relative to the ground result in quite large errors in the strikes. The purpose of our study was to find a method of eliminating these errors, for, of course, every bomb that is dropped in the wrong spot is not only a waste in itself but is a waste of the time, personnel, and equipment used in making the bombing mission.

The problem may be visualized if one imagines that one looks down on a bomb that is falling from an airplane. Before the bomb is dropped the plane has been flying in a straight line toward the target for some time. This portion of its flight is called the bombing run. If the flight

of the plane is continued in the same direction as the bombing run after the bomb has been dropped, the bomb seems to remain approximately vertically under the plane and stays that way until it hits the ground. If one draws an imaginary line through the bomb in the apparent direction of its flight, it is easy to see whether it is going to the right or to the left of the target. One can then imagine at once a possibility of doing some correcting of any bombardier's error by steering the bomb to the right or left. It is not possible by looking at such a bomb to tell whether it is going to hit the ground too soon or too late to land on the target. For this purpose it is necessary to use a special maneuver or special apparatus. The special maneuver was unsatisfactory and the special apparatus which was developed at the Franklin Institute was used.

The analysis of bombing experience in combat reveals that to correct bombardiers' errors completely it is necessary not only to correct the direction, or, as we often say, the azimuth of the flight of the bomb, but also to correct its range. We can, therefore, consider the problem as consisting of two parts, the right and left deflection and the change in range. The former is controlled by means of rudders and the latter may be controlled by means of elevators.

To control the flight of a bomb, it is necessary to keep it from rotating about its axis. Otherwise rudders would sometimes be elevators and vice-versa, and control would be impossible. The first method that suggests itself for eliminating this rotation or roll is to provide stabilization by means of a gyro. The simplest way to apply these controls is by means of ailerons, a name which we borrow from aeronautics. These control surfaces are so arranged that when they are operated by the gyroscopes they produce a rotation of the bomb about its axis but no yaw or pitch.

If a bomb is to be controlled from an airplane, it is necessary also to provide in some way a means of communication between the airplane and the bomb to transmit the controls. Radio is the obvious solution and is the one that was used in our development.

To give an idea of the course the development necessarily had to take, a few of the early questions may be enumerated.

(1) Is it possible to control the high angle bomb sufficiently to correct the errors that are likely to be encountered? The errors from 15,000 ft. were of the order of a few feet to several hundred, or even occasionally a thousand feet.

(2) What kind of aerodynamic surfaces must be used in order to get this control?

(3) What physical shape should the controlled bomb take?

(4) What is the behavior of bombs in their flight to the ground? Do they all rotate or do they very seldom rotate?

Many control systems were proposed and during the first two or three years of the development all possibilities were followed. One of these was the use of television equipment in the bomb. This was arranged to transmit to the operator a picture of what could be seen from the nose of the bomb itself. If this picture had the target in the center it was an indication that the course would carry it to the target. If the target was not in the center, application of the control from the plane would correct the course of the bomb.

Another promising type of apparatus made use of target seeking equipment and was operated by radiations, either of light or heat. The action was automatic in that the equipment was arranged so that the heat sensitive or the light sensitive equipment would initiate controls to keep the target in the center of the bomb's field of view.

Both of these methods of attack had to be abandoned because the apparatus was too bulky, and too complicated to be perfected in a short time. There remained the direct sight method, and since that was the one finally used, it will be discussed more in detail.

To establish the possibility of getting effective controls, the earliest experiments were made with small models in the wind tunnels at M.I.T. The wind tunnel experiments indicated that the controls would be adequate. In order to get verification of the results obtained in the wind tunnel and of the very large extrapolation that was necessary, 100 lb. practice bombs were taken to Aberdeen Proving Grounds, and tested to determine their behavior in the air. For these experiments small rudders were built into the tail structure of the practice bomb, with the idea of determining how large were the effects of the rudder surfaces. In these experiments it was soon noticed that the bombs rotated so much that it was difficult to get satisfactory estimates of the effects of the surfaces. Occasionally, however, the rotation was small enough to give promising indications. It was necessary to make observation from the ground because the men in the plane were not able to see the the bombs throughout the entire flight. In spite of all the difficulties and shortcomings in the early experimental methods, something was learned about the size of rudders that were needed and everyone was convinced that gyro stabilization was absolutely necessary. It was learned also that if we were to think of using direct sight in our control system, it would be necessary to provide the bombs with flares.

Television and direct sight bombs were now under construction at Massachusetts Institute of Technology. All essential apparatus including gyro equipment, radio, and the servo mechanism had been initiated and some of it was under construction.

At about this stage of the development, the Gulf Research and Development Corporation came into the project under a contract with the National Defense Research Committee. It was realized that a better method of experimentation in the field was necessary. Motion

picture cameras were mounted in the nose of the bomb and recorded what the bomb saw during its fall. Clockwork mechanisms were provided and were pre-set to apply controls for predetermined periods during the drop. The bomb was "exploded" near the end of its flight so that the various parts of apparatus were carried to earth on parachutes.

As soon as an adequate flare was made available, another experimental method was used very extensively, namely to take motion pictures of the bomb as seen from the plane. The motion pictures taken from the plane and those taken from the nose of the bomb could be combined to provide material for a very satisfactory analysis of the actual trajectory. These analyses showed how the bomb responds to the control surfaces, how large the control surfaces needed to be to get a certain over-all result, what angle of deflection was necessary, how the bomb responded to aileron control, how much yaw or pitch was involved in the application of controls, the period of oscillation of the bomb both in yaw or pitch and in rotation, the rate at which the bomb responds to control and many other details that were necessary for the solution of the problem.

Early tests with television and with direct vision controls in azimuth and range (Razon), and also in azimuth only (Azon) were made at Eglin Field, Florida. By this time the essential elements of the bomb control equipment had all been included and some study of the aerodynamics had been made by the workers at M.I.T. The completion of the development of practical experimental units, both Azon and Razon, was done by the Gulf Research and Development Corporation, as was also the extensive and elaborate field tests, wind tunnel tests, analyses and calculations of trajectories that followed and that were necessary to determine aerodynamic and other characteristics.

A primary component part of the bomb equipment in direct sight missiles is the radio receiver, which in the experimental units consisted of a super-regenerative radio amplifier arranged to select functions by means of a code.

The gyro equipment consisted of two gyroscopes, one called a free or directional gyro and the other a rate gyro. The free gyro was used to return the bomb from any position at which the rudders were not vertical and the rate gyro was used to keep the speed of rotation of the bomb below a certain value. It was found that if the directional gyro were used alone, the bomb would return to its normal position with such high angular velocity that it would over-shoot and go into violent oscillations. With the rate gyro cooperating it was possible to bring the the bomb back to its normal position at a reduced speed so that it would not over-shoot by more than about three degrees and immediately settle down into its normal position. This gyroscope assembly was developed in its original form by M.I.T. and was very much improved

by the Gulf Laboratories. In its experimental form the gyroscopes were driven by air, being connected to the vacuum system of the plane.

At this stage it was realized that it would not be possible in a reasonable length of time to develop all the bombs that had been started. The Chief of Division 5, N.D.R.C., therefore called a meeting in February 1943 to discuss this question and to decide which method of attack should be used. The meeting was attended by many representatives of the Army Air Forces and all the members of Division 5 and Section 5.2 N.D.R.C. The Chief of Section 5.2 recommended that for the present we concentrate our attention on the direct vision method of control and apply it to azimuth only. He was alone in his argument

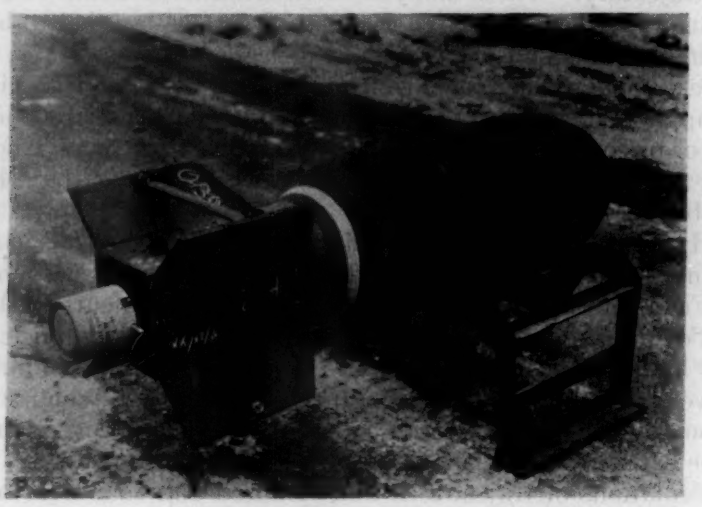

FIG. 1. 1000 pound Azon (VB-1).

for this program and the Air Force representatives and the aerodynamicists present were pessimistic about the possibility of getting adequate control by the method that was suggested. After a long discussion it was decided that the work on the direct vision bomb should be discontinued and the work on the homing bombs should be pushed. The reason for this was that it was thought humanly impossible to guide a missile that travels as fast as a high angle bomb does during the last part of its flight. On the other hand it was believed that if the apparatus could be made to report automatically to the control equipment, all of which is in the bomb itself, the results would be more easily attained.

Following this meeting and while preparations were being made for a more complete test of target seeking bombs, the Azon tests were continued in order to get additional aerodynamic data. Two months later, after having tested 13 Azons, there had been obtained some very satis-

factory movies taken from the airplane showing the performance of an Azon in flight, and a strike on a road target 12 ft. wide. Bomb No. 13 in this series was controlled all the way to the ground, and the motion pictures showed that by watching the flare it was very easy to tell when the bomb was to the right or to the left of the road, and the effects of application of control were very evident. It had been found that from 15,000 ft. the bomb could be deflected from its normal path by 2,500 to 3,000 ft. The motion picture was taken to Washington and shown to

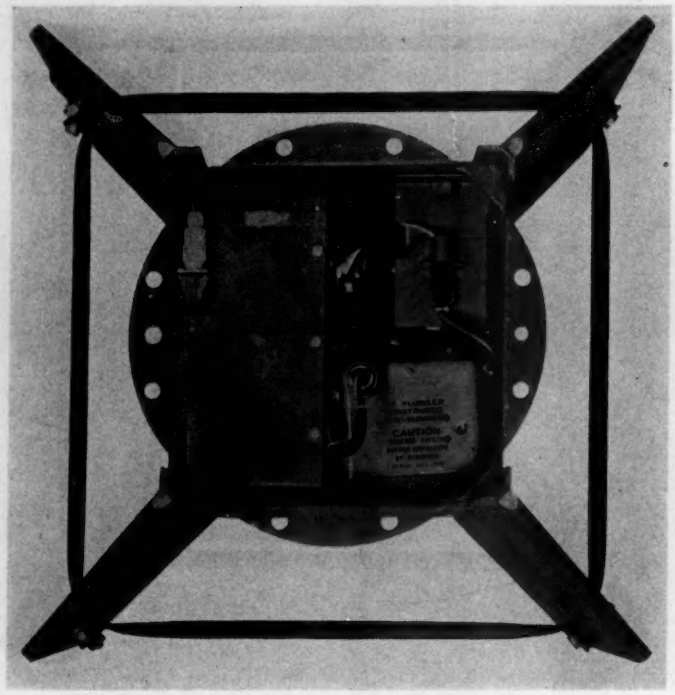

FIG. 2. Leading end of tail structure showing radio receiver, gyro, and battery.

a number of interested Army officers in the Pentagon building. After seeing this performance, there was a complete reversal of opinion and a demand for a crash program to get Azon into production. Azon had been recommended for use on roads and bridges, and it was obvious from the experiment that it would be a very effective weapon against such targets.

The engineering and production was assigned to the Union Switch and Signal Company, and the changes from the experimental unit that were necessary for production and for use in the field may be briefly stated as follows:

(1) The body of the fin structure was changed from a round section to a square section and to welded construction (Fig. 1).

(2) The gyroscope unit was changed from air-drive vacuum-operated to electric-drive (Fig. 2).

(3) The battery was changed from 12 volts to a smaller 24-volt battery using the NT-6 battery which had been developed by the Willard Battery Company for the Navy for another purpose.

(4) Solenoids for operating the ailerons were changed to eliminate much mechanical linkage and to provide individual adjustment (Fig. 3).

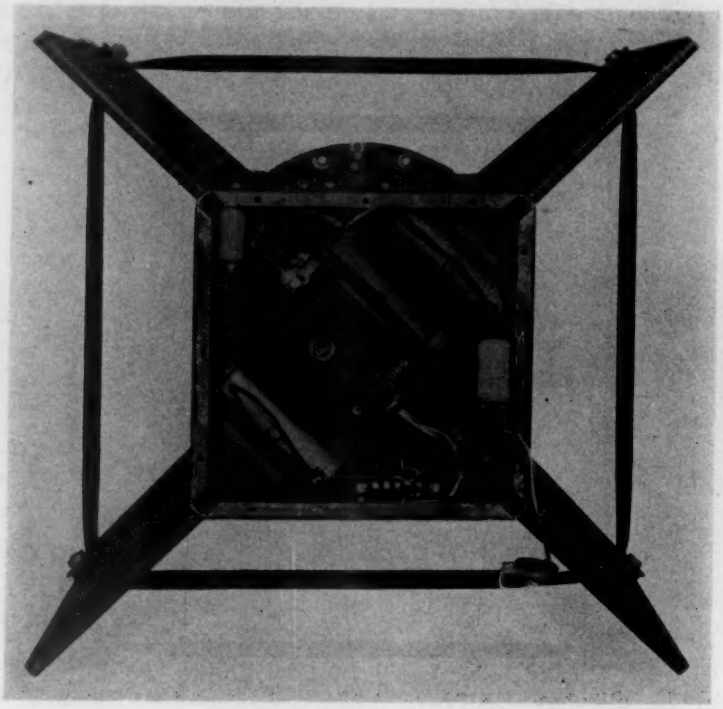

FIG. 3. Trailing end of tail structure showing solenoids and servo motors.

(5) Radio. In the production units the selection of functions was made by means of audio frequency modulations instead of by means of the long and short pulses. The receiver was later changed to a superheterodyne.

(6) The struts which were installed to stiffen the fins were used also as antennas. They were streamlined and insulated and were brought inside the circumscribed square.

(7) The servo motors were wound for 24 volts.

(8) The circuits were modified by adding a kick-out switch through which the equipment could be warmed up and through which the battery could be charged during flight, and by means of which all the circuits could be transferred to the bomb's battery when the bomb was

released. Flare ignition was added to the electrical circuit and the ignition was made by means of a squib instead of with a match. A protective relay was added which made it impossible to ignite the flare until the bombardier had thrown a switch, which was done any time after take-off. An electric gyro release was added to the circuits.

(9) The flare was perfected by the Army Ordnance Department.

(10) The disposition of the various components in the tail structure was changed so that they were all accessible for removal.

One hundred and fifty units of this type were built on a pilot production order. The first of these were delivered November 25, 1943 and were immediately tested at Eglin Field. Many of the units from the pilot production were used in combat.

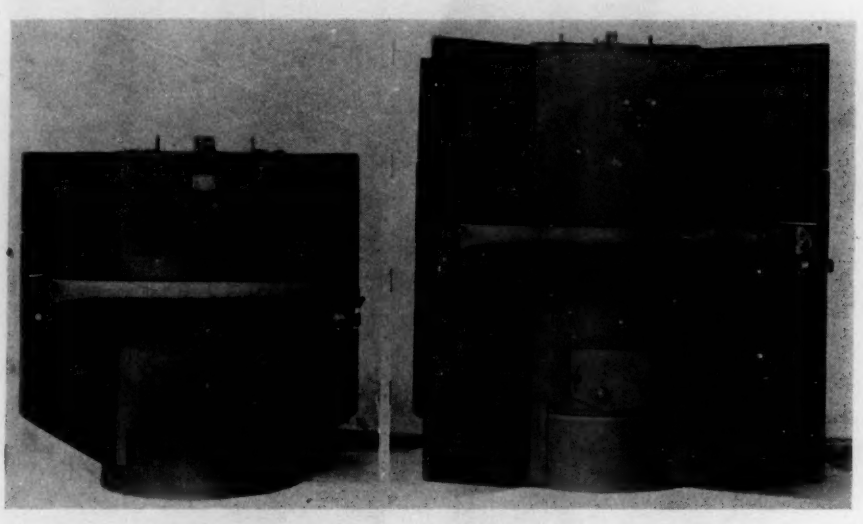

FIG. 4. Relative sizes of a 1000 pound and a 2000 pound (VB-2) Azon tail structure.

During the work on the 1,000 lb. Azon, VB-1, the development had also been completed of a 2,000 lb. Azon, VB-2 (Fig. 4). This unit was given a thorough test by Division 5 and the Air Forces, but was not used in combat.

During and following the work on VB-1, the development of the Razon was completed by the Gulf Research Laboratories. It had been started at M.I.T.

In the case of the Razon it was thought wise to go to a different type of tail construction because of the possibility that the cruciform fin structure had a tendency to produce rotation if the bombs were subjected simultaneously to yaw and pitch, and because of the necessity of getting something that was definitely practical at the earliest possible time. The cruciform structure was not finally abandoned, but was set

aside for the moment on account of the possibility that it might require a more extensive development. The structure that was finally adopted for the Razon, known to the Army as VB-3, had aerodynamic surfaces in the form of two shrouds (Fig. 5). One shroud was mounted at the edge of the structure next to the bomb. The second shroud was mounted with its trailing edge flush with the rear edge of the box part

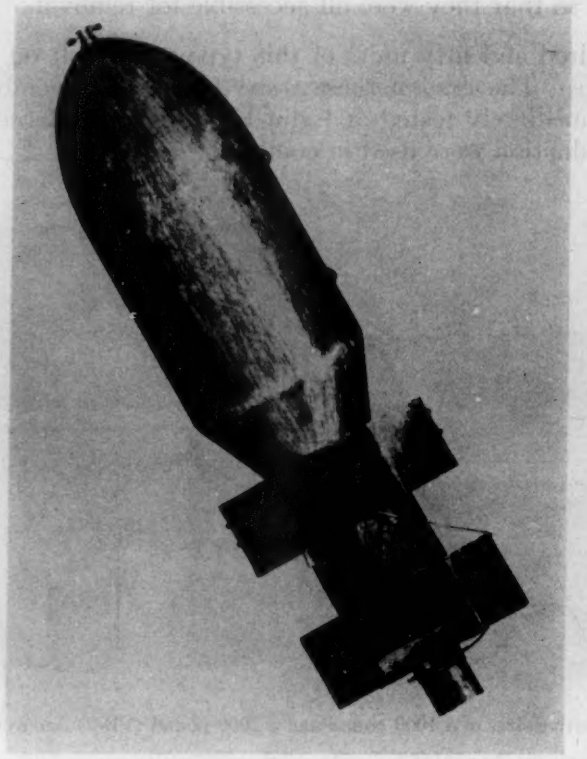

FIG. 5. Artist's drawing of 1000 pound Razon (VB-3).

of the fin structure. The former shroud may be called a lift shroud and the latter a control shroud.

The control surfaces were built as flaps on the trailing edges of the long sides of the control shroud. The ailerons were built into the struts supporting the control shroud and were four in number. Except for the changes necessary to control in two directions, the apparatus contained in the apparatus compartment was the same as in the Azon (Fig. 6).

The development of the Razon was completed as were also the field tests. After providing certain corrections that were necessary in the

use of the sighting equipment, the errors in drops from an altitude of 15,000 ft. were reduced to approximately 45 ft. in range and 10 to 15 ft. in azimuth. The Razon was not used in combat. At the end of the war it was in production at the plant of the Union Switch and Signal Company, and 3000 units were made for experimental use by the Army Air Forces.

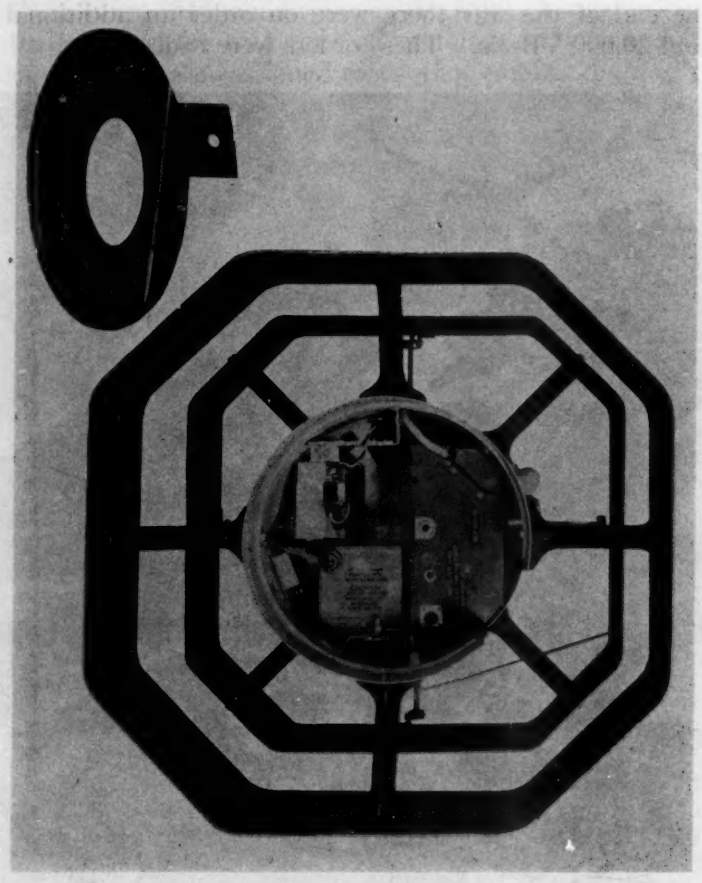

FIG. 6. Production model of VB-3 apparatus showing shrouds, radio, gyro, and battery.

Since the components that had been developed for the Azon (Known as VB-1 and VB-2) and their method of mounting were also used in the experimental Razon (known as VB-3), very little extra engineering was required for production.

(In the oral presentation there were shown moving pictures of tests and combat performance of the bombs, and a demonstration of the Azon apparatus.)

The total construction of high angle guided missiles by the Union Switch and Signal Company was as follows:

VB-1	15,271
VB-2	132
VB-3	3,150
Total	18,553

At the end of the war there were on order an additional 10,000 VB-1's and 20,000 VB-3's. These orders were cancelled.

FIG. 7. Azon hits on a bridge in Burma theatre.

The first delivery of regular production models was made March 27, 1944.

A squadron of B-17's took the Azon into combat in February, 1944, in Italy, three and one-half years after the initiation of the research work. These were pilot production units and the 15th Air Force made

the first applications. During the Italian campaign some bridges were destroyed, and at one time the viaduct leading to the Brenner Pass was put out of service.

The next use of the Azon was made in France by the 8th Air Force operating from England. Here also a number of bridges were destroyed, specifically bridges on the Loire and the Seine, and the members of the group that were using the Azon were enthusiastic, but in the latter half of June, 1944, conditions at the front changed so that there was no further need for that type of bombing, and the unit that had been using the Azon was assigned to another combat duty.

The really effective use of the Azon was made in Burma by the 10th Air Force. Here there were not only adequate equipment and trained crews, but also Army representatives and civilian representatives who were well trained in the use of Azons and who knew their capabilities. Results in Burma were very gratifying. Approximately 150 bridges were destroyed. Sometimes a considerable number were destroyed in one mission. The result was a complete disruption of the Japanese system of communications in this theatre.

The increase in effectiveness of high angle bombing as applied to long targets that results from the use of Azons instead of standard bombs is illustrated by the fact that the number of bombs required to destroy a bridge was reduced from 300 or 400 of the standard bombs to six or seven Azons. Illustration of the effectiveness is shown in Fig. 7. In this case four planes dropped one Azon and one standard bomb apiece, each bombardier steering his own Azon, and it will be seen that of the four Azons three hit the bridge. The fourth landed near the end of the bridge. The standard bombs may be seen at various considerable distances from the target.

To have had a part in such a development was a great privilege, the opportunity to tell you about it has been a great pleasure and honor, and the interest that you have shown has been very gratifying and much appreciated.

Thank you.

Picture, Made at Millionth of Second Exposure, Developed for Projection in 30 Seconds.—A camera with a speed of one-millionth of a second which produces a finished photographic projection within 30 seconds after the picture is taken has been developed by General Electric's General Engineering and Consulting Laboratory.

The camera, being two feet long and one foot high and deep, is not portable, and is not intended for use by the camera fan. Fully automatic with the press of a button, it is part of equipment developed for the rapid testing, by means of electric power surges, of apparatus used in the generation and transmission of electric power. Such tests are made to determine insulation characteristics of new designs, and to assure that equipment under production meets performance specifications.

Used in combination with a cathode ray oscillograph, the camera photographs a visual indication which appears on the television-like screen when a surge of high-voltage electric power is applied to the equipment under test. Since the total duration of the recorded voltage wave may be as short as one millionth to a few hundred millionths of a second, the camera speed must be exceptionally high, according to N. Rohats, G-E engineer who directed work on the test equipment.

As soon as the photograph is taken, the operator pushes a button, thereby setting into action the automatic developing equipment built into the camera. The development cycle is finished in 24 seconds, and the film is moved into another compartment in which a projector reproduces the negative, enlarged about ten times, on a ground-glass screen at one side of the camera. This picture gives the operator an accurate record of the performance of the equipment during the application of the surge.

Mr. Rohats pointed out that the time saved by the high-speed camera is significant because of the large investment involved in the equipment being tested, the oscillograph, and the associated test set, a generator which produces surges of up to 3,000,000 volts. With this new equipment, testing of electric apparatus will be speeded up as much as ten times, he estimated.

R. H. O.

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ULTRASONIC DELAY LINES II.*

BY

A. G. EMSLIE,# H. B. HUNTINGTON,‡ H. SHAPIRO,§ and A. E. BENFIELD,†

Massachusetts Institute of Technology, Cambridge, Massachusetts.

ABSTRACT.

This paper deals with the experimental aspects of ultrasonic delay lines in liquid media and is a sequel to a previous paper treating the general theory of such devices.

Delay devices employing ultrasonic propagation through liquids have proved satisfactory for delay times of the order of 3 milliseconds or less. Such devices can be made broadband and give excellent reproduction of pulse shape. The design considerations for such devices are discussed and illustrated by actual examples.

A. INTRODUCTION.

In an earlier paper[1] it was pointed out that electrical signals can be satisfactorily delayed for a time as long as a few milliseconds by ultrasonic techniques. The electrical signal is transformed by a transducer, such as a quartz crystal, into an ultrasonic signal that follows a fixed path through a transmitting medium to a second transducer which, in turn, reconverts the pressure pattern into an electrical signal. This part of the report deals with the use of such delays in liquid media at Radiation Laboratory. (Considerable success was also obtained using solid materials for ultrasonic delays but this phase will not be treated here.)

By way of historical background, it should be pointed out that the first liquid delay line, at least in any widespread application, was the Shockley delay unit for the production of a variable, calibrated trigger. Quartz crystals were employed for transducers. The transmitting medium was a mixture of water and ethylene glycol held at a constant temperature of 55°C. to prevent velocity fluctuations. The unit was developed and engineered by the Bell Telephone Laboratories and was in production by the end of 1942.

A second ultrasonic device using mercury for the transmitting medium was investigated independently by the Moore School of Electrical

* This paper is based on work done for the Office of Scientific Research and Development under Contract OEMsr-262 with the Massachusetts Institute of Technology.

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§ Now at The National Institute of Health, Bethesda, Maryland.

† Now at Harvard University, Cambridge, Massachusetts.

[1] H. B. Huntington, A. G. Emslie, and V. W. Hughes. "Ultrasonic Delay Lines I," JOURNAL FRANKLIN INST., Vol. 245, No. 1, January, 1948, p. 1. Henceforth this paper will be referred to as I.

Engineering at the University of Pennsylvania during the summer of 1943. Ultrasonic delays, particularly for use in radar, were subsequently employed in this country, England and Germany.

Radar systems, designed to employ pulse-to-pulse cancellation, had need for a means of faithfully delaying microsecond pulses for times of the order of 10^{-3} sec. In England, H. Grayson at TRE developed a water line satisfactory for this purpose. At Radiation Laboratory, work was initiated around the end of 1943 to investigate the possibilities of a mercury line for a similar use. In the course of time, as various projects presented their own specifications, each conditioned by their particular problems, the work broke up into separate programs and several small groups of individuals went ahead on more or less parallel lines of development.

The original research was carried out by G. D. Forbes and H. Shapiro. Mercury delay lines, built according to their design, gave very faithful pulse reproduction at the required delay items of the order 500 to 1,000 microseconds. These units were incorporated into an experimental radar system where their successful performance gave a great impetus to the whole program. Forbes and Shapiro investigated other possibilities such as solid rods, narrow tubes, etc. Shapiro continued with research on mercury tanks and on transmission through wires while Forbes later collaborated with P.R. Bell in designing a mercury delay line for a radar system.

Following the successful experimental system several different radar systems were developed which incorporated the ultrasonic mercury delay line as an essential component. The units had to be engineered to withstand a wide variety of field conditions and were installed in trucks, planes and ships. The length of the temporal delay varied from one application to another from roughly 500 to 3,500 microseconds. The length of delay had in turn a bearing on the choice of carrier frequency. All this required considerable development and some research. As an example of the latter, it became important to measure the velocity of mercury as a function of temperature to a high degree of accuracy. These measurements were performed by R. I. Jacobson.[2]

In general a well-designed delay line should possess the following characteristics: (a) rugged mechanical construction, (b) accurate preservation of pulse shape, which requires adequate bandwidth, (c) no accessory signals, (d) moderate attenuation, and, for some applications, (e) accurately controlled delay.

B. DEVELOPMENTS AND TECHNIQUES.

1. Capillaries and Wires.

Much of the early research was directed toward using capillaries or wires to transmit the supersonic pulses but these efforts met with

[2] Radiation Laboratory Report 745: "Measurement of Supersonic Velocity in Mercury at 15 Megacycles per Second as a Function of Temperature."

very little success. As the saving of space was always desirable and particularly so in the case of the longer delays, the possibility of using a coiled delay line appeared an inviting prospect. It is known from work at lower frequencies that it is possible to send an acoustic beam around a moderately curved tube if the wave length is comparable with tube diameter. For this reason, it was natural to try mercury in a curved capillary. It was found that the pulse could traverse several bends in such a capillary with little distortion, but that the attenuation was excessive.

Properties of thin rods and wires when used to transmit supersonics will be only briefly mentioned here. Transmission through solid rods involves many difficulties. One of the most troublesome effects is the appearance of accessory signals caused by the excitation of transverse and surface modes at the surface of the rod. This effect can be reduced by roughening the sides of the rod or increasing the diameter. If, on the other hand, one decreases the diameter with a view to approaching the condition of the coiled wire, attenuation increases and pulse shape becomes distorted.

2. Mechanical Construction and Assembly Technique for the Laboratory Delay Line.

The delay lines built for the original experimental system were successful in providing broad bandwidth, faithful waveform reproduction, accurate alignment and freedom from accessory signals for delays of the order of 500 microseconds at a carrier frequency of 15 Mc. In addition, no deterioration of their performance was noticed with time.

To achieve the required bandwidth mercury was used for the transmitting medium and cells filled with mercury were used as electrodes in back of the crystals. The high acoustic impedance of the mercury broadens the crystal's response, as has already been shown (I, Sec. B). Many of the original lines were constructed from textolite, since it was easily machined. An example is depicted in Fig. 1, which shows clearly the mercury cells backing the crystals.

The purpose of these cells was to eliminate accessory signals which would otherwise arise from multiple reflections at the crystal surfaces. Since the crystals were usually used at resonance, they acted as half wave sections of nearly lossless transmission line. Consequently, for the same material on both sides of the crystal there was practically no impedance mismatch and the supersonic beam was transmitted into the end cell. There the beam struck the end glass plate and was completely dispersed by multiple reflections inside the end cell so that no fraction could emerge in phase to give an accessory signal at a later time.

Because of its superior mechanical and electrical properties, cold rolled steel tubing was used in constructing later models. The impedance mismatch between the quartz and the lacquer layer employed to

make it adhere to the steel appeared to be sufficient so that no perceptible accessory signal was transmitted through the steel. The alignment of the crystals at the end of the main tube was facilitated if the ends were machined parallel to each other. This could be done with a parallelism of better than several hundredths of a degree in a good lathe by placing arbors in the ends of the line and mounting the tube accurately on the lathe centers. With the cutting tool one end of the tube was faced off and then, without removing the tube from the lathe, or readjusting its position, the tool was moved to the other end of the tube to face that end also.

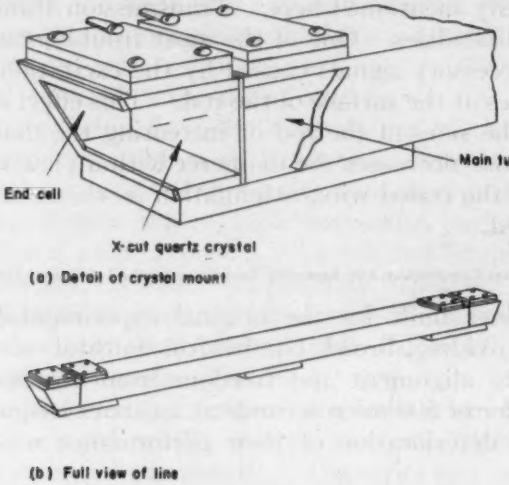

FIG. 1. Liquid delay line for experimental system.

Cleaning is an important preliminary to the assemply of mercury lines as scum caused by dirt or grease in the mercury will adversely affect performance by causing excessive attenuation. Material delivered from the shop was carefully cleaned by immersion in or filling with some organic grease solvent. The opertion was to be repeated several times, the final rinse being done with unused solvent.

The crystals used in this work are usually thinner than 0.3 mm. in thickness and some care is required in their handling. They are generally waxed to plate glass by their manufacturer before delivery. A gentle heating will suffice to melt the wax and the crystal can then be slid off. Before mounting it should be cleaned carefully, again with grease solvent, to remove all wax and dirt.

To assemble a steel line (see Fig. 2), one crystal was lacquered in place across the end of the tube and the lacquer allowed to harden. The second crystal was then applied to the other end and, while the lacquer was still hardening, small adjustments could be made to improve the alignment. The accuracy of the alignment could be accurately

checked by observing how closely a point object lined up with its optical images from both crystals. Next the end cells were similarly lacquered at the end and applied in place to the crystals. Four bakelite straps held the end cell to the main tube. To maintain the rigidity of the

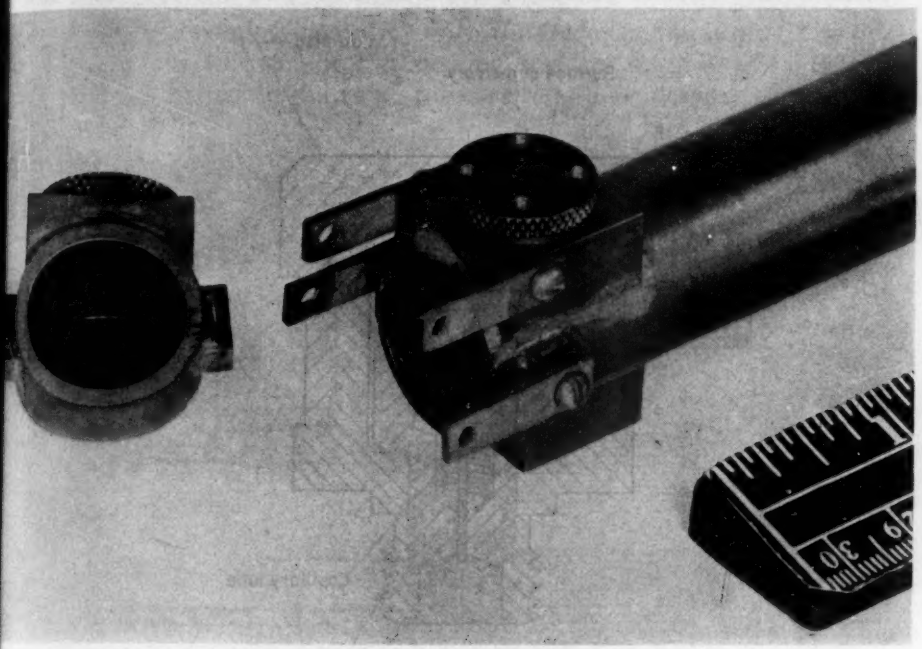

FIG. 2. Delay line before assembly.

line, it was mounted on a flat length of steel channel which had previously been strain-relieved by heat treatment.

3. The Elimination of Accessory Signals.

Though the mercury end cell described in the preceding subsection proved to be satisfactory for a laboratory delay line or use in an experimental system, the quartz crystal supported by mercury on both sides was liable to fracture under shock. When the crystal mounting with absorbing backing was put into field use, some provision had to be made to prevent transient differentials in pressure from breaking the crystal. If the fluid on both sides of the crystal completely filled its container so that there was no air space into which it could be driven by sudden shock, equality of pressure would be maintained under mechanical test but not under thermal variation. The problem was to a large extent solved by inserting a coiled, steel capillary (Fig. 3), which acted as a connecting link between the completely filled chamber in the line and a partially filled chamber in the loading plug. The short time transients

were damped by the viscous forces in the capillary walls but slow thermal variations altered the mercury level in the cap chamber and produced no net pressure change.

Later some modifications in absorbing backings for crystals were introduced. A steel backing piece was used with a deeply cut, saw-

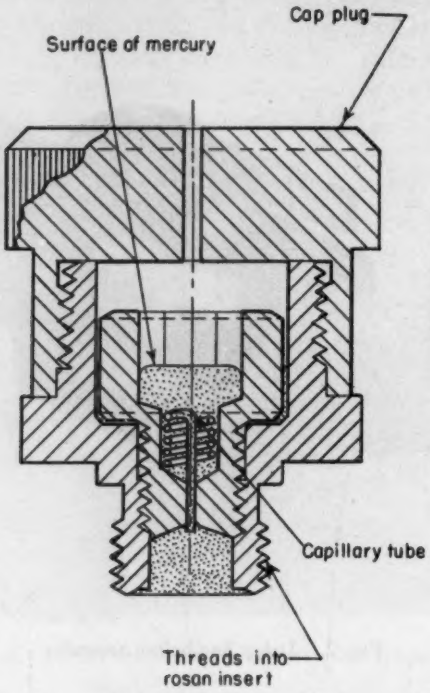

FIG. 3. Mercury inlet with capillary loading.

toothed surface (Fig. 4). The tops of the sawteeth supported the crystal's back surface and the space between the teeth was filled with mercury. The supporting steel area should be too small to give appreciable reflection and still support the crystal adequately under variations of static pressure as well as sudden shock.

There is another promising possibility for an absorbing backing which has not yet been incorporated in any system application, namely, to use a crystal soldered to a solid material which matches well the transmitting medium. For mercury a good acoustic match can be obtained with a backing of hard lead (6 per cent antimony). Preliminary measurements have shown that the reflected signal is reduced by some 18 db under these conditions as compared to the case when the crystal is supported by a dry electrode. The material has the advantages that it is machinable and readily attenuates the transmitted energy. For soldering a metal to quartz, a metallic film must first be

deposited by evaporation on the quartz. Nickel is used for the first few layers nearest the glass, usually followed by metals of higher vapor pressure. Once the crystal has been soldered to the lead electrode, the latter can be cemented into a dielectric support and incorporated into the end assembly. Some precaution should be taken to prevent the mercury from coming in contact with the solder or the lead. The final step would be to machine the aligning surfaces so that when the end assembly is bolted to the tube, the crystal will be perpendicular to the

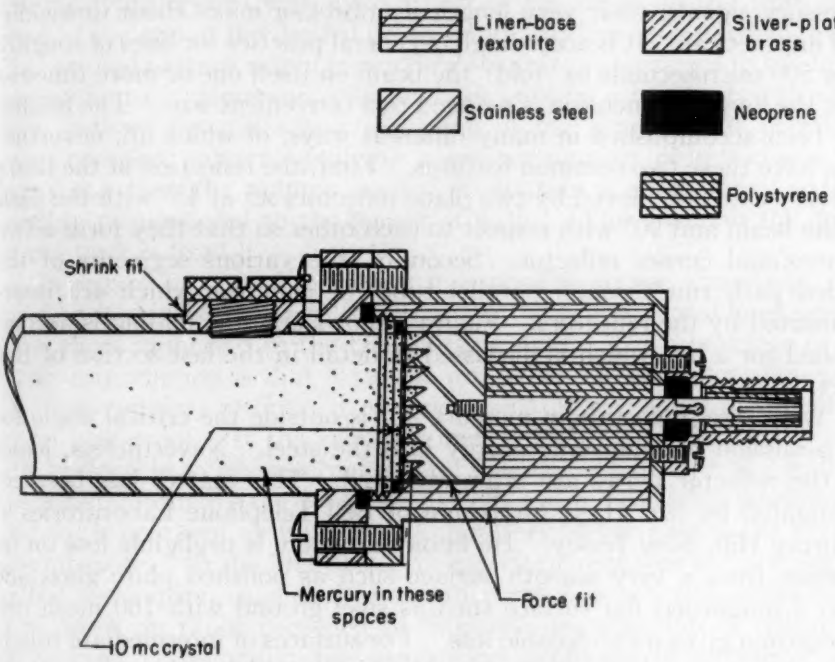

FIG. 4. End assembly with sawtooth backing.

tube axis. For those projects which required longer delays, 1,000 microseconds more, there was little need for the crystal mounting to absorb the incident energy. The attenuation in transit was sufficient in these cases to obliterate any accessory signals that might have been caused by multiple reflections. This meant that the crystal could be supported by dry metal electrodes which give rise to nearly total reflection. The acoustic impedance of air is so small that even the thin layer that exists between the metal plate and the quartz is sufficient to supply nearly complete mismatch. Since it was considerably simpler to support the crystal with mountings of this sort, it became general practice to make use of them wherever possible. To a certain extent, it was possible to control the attenuation loss by transit through the medium by appropriate choices of the carrier frequency and of the tube

size. It has been pointed out in I, Sec. D, that the free-space attenuation in a liquid medium varies as the square of frequency of propagation. There are also losses from the diffraction spreading of the beam and, in a tube, losses possibly arise from the viscous forces at the walls. Such losses depend on the size of the crystals and the inner cross section of the tube.

4. Folded Lines.

Though the longer lines simplify the problem of eliminating the accessory signals, their very length if unbroken make them unwieldly and impractical. It is accordingly a general practice for lines of roughly over 500 microseconds to "fold" the beam on itself one or more times so that the longest dimension is reduced to a convenient size. The folding has been accomplished in many different ways, of which all, nevertheless, have these two common features. First, the reflection of the beam in every case is achieved by two plane reflectors set at 45° with the path of the beam and 90° with respect to each other so that they form a two dimensional corner reflector. Secondly, the various segments of the folded path run through parallel tubes or channels which are interconnected by the reflectors. Figure 8 shows how the folding is accomplished for a line which is discussed in detail in the last section of this part.

With steel reflectors in mercury 45° is outside the critical angle for transmission of supersonic energy into the steel. Nevertheless, losses at the reflector plates are often observed. This matter has been investigated by Mr. H. J. McSkimin of Bell Telephone Laboratories at Murray Hill, New Jersey. He finds that there is negligible loss on reflection from a very smooth surface such as polished plate glass and also a roughened flat surface such as steel ground with 160 mesh carborundum gives no noticeable loss. For surfaces of intermediate roughness, however, a loss can occur. McSkimin explains this effect as the results of phase incoherence in the reflected beam. Since mercury does not wet surfaces with which it does not form an amalgam, small bubbles of air will lodge between the mercury and the steel at any irregularities in the steel surface. That portion of the compressional beam which is reflected from the air surfaces will be reflected with 180° change of phase with respect to the incident beam. The phase change of the part of the beam reflected from the steel had been calculated to be about 80° with respect to the incident beam. If the parts of the beam are nearly equal, then one might expect a loss due to phase incoherence as high as 3 db per reflection. For the very rough surface the reflection is practically all from the air and consequently the phase incoherence is small. McSkimin recommends this surface for actual reflectors as dirt and scum has less effect on this surface than on the polished smooth surface where the reflection is primarily from the solid material of the reflector.

5. Tanks.

The search for a method to package the supersonic beam even more compactly than is done in the folded lines led, naturally, to the investigation of liquid tanks in which the beam could be multiply reflected from the tank walls. The design on paper of such a device proves to be rather an amusing pastime which appears to call for some ingenuity on the part of the designer. Actually with any scheme, one is fundamentally limited by spreading of the beam, since it is important to avoid striking the receiving crystal with even a part of the beam at any time other than at the end of the desired delay time. Scaling up a particular two-dimensional pattern would give a delay that is proportional to the linear scaling factor. However, a larger tank permits a larger crystal which then reduces the diffraction spreading of the beam. This permits a more complex pattern involving a greater number or reflections. It turns out that the volume needed to contain a given delay path is roughly proportional to the length of path. (This assumes the depth of the tank is fixed.)

In regard to crystal mounting it is possible to use absorbing, mercury-backed cells similar to those used in the experimental lines, and to have them immersed in the transmitting fluid. The advantage of the latter construction is that it can be rotated to obtain exact alignment. Its disadvantages are that it occupies valuable space and is difficult to make leak-proof. If the required mechanical tolerances can be met, it is preferable to use external crystal assemblies bolted to the walls of the tank. Actually the tanks do not offer very promising possibilities and work on them at Radiation Laboratory never passed the experimental stage. In the first place, their performance was not predictable as the mechanism causing loss at reflection was not well understood. (The work on tanks preceded McSkimin's investigation of reflections in mercury.) Secondly, the mechanical problems in alignment and leak-proof construction appeared formidable. Lastly, the net gain in space and weight did not appear very considerable. It seems that by increasing the number of corner reflector and parallel channels, one could use the folded line technique to get within a factor or two of the space required by a tank. With most liquids it is true there would be a considerable saving in weight, but with mercury the tank would probably be heavier than the folded line.

If one should, nevertheless, decide to construct a mercury delay tank on the basis of what is now known something like the design in Fig. 5 might serve as a basis. The tank would be rectangular with inner dimensions na by $(n + 1)a$. The total delay path would be $\sqrt{2}\,a\,[n(n + 1) - 1]$ and the final beam spreading should be considerably less than $a/\sqrt{2}$. All reflections would be at 45° and should give no serious loss if the walls are flat and roughened. The depth of the tank

in most cases should not greatly exceed the diameter of the crystals, as this would help decrease the loss from beam spreading.

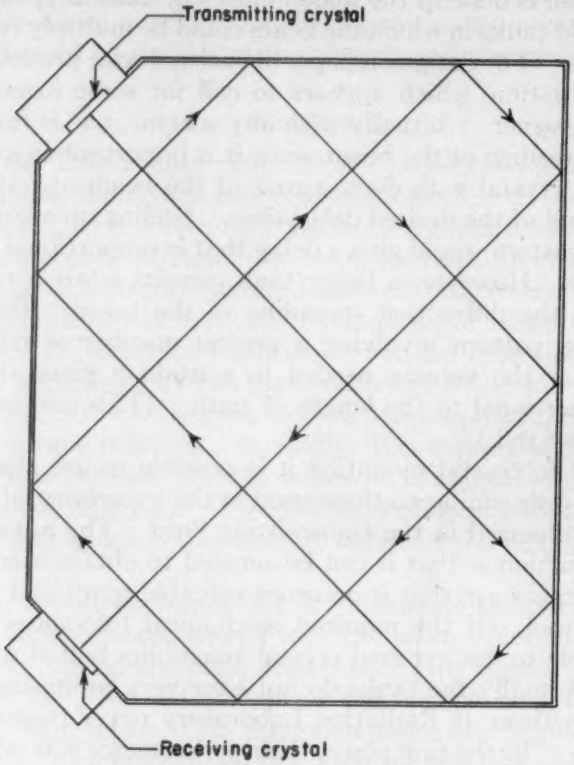

FIG. 5. Design for liquid delay tank.

C. DESIGN CONSIDERATIONS.

If one is asked to construct a line which is to delay pulses of a given duration and rise-time for a given time, one has the following design parameters at his disposal: transmitting medium, carrier frequency, crystal fundamental frequency, crystal and tube size, number of channels and reflectors, and required mechanical tolerance. An attempt will be made to discuss the factors which influence the choice of the various parameters and to point out the interrelations involved.

In choosing the transmitting medium one is limited to those with moderately low attenuation. Highly viscous liquids are immediately ruled out.

The next consideration is the elimination of beam distortion arising from thermal gradients in the medium, since the acoustic velocity is generally a function of temperature. Little trouble is experienced with mercury since its heat conductivity is high and its thermal coeffi-

cient of velocity rather low, about 3 parts in 10^{-4}. With water and most other liquids, the effect is very serious, unless the medium is held at a temperature for which the thermal coefficient of velocity vanishes. Measurements [3] at the Bell Telephone Laboratories have shown that the acoustic velocity in water goes through a maximum for 74° C. and that many water mixtures have similar maxima at lower temperatures. Such media held at their temperature of maximum velocity will give constant delay. In addition, the attenuation of water decreases with rising temperature.[4]

Finally, the acoustic impedance of the transmitting medium has an important influence on the pass band of the transducer (see I, Sec. B). For applications requiring broad bandwidth the large acoustic impedance of mercury gives an immediate solution. It is possible, however, to obtain comparable bandwidth with a medium of low acoustic impedance if the crystal is backed by a material of high acoustic impedance, such as mercury or a soldered metal electrode. One obtains in this case a broad, double-humped passband (see Fig. 7, Part I) which is not likely to cause much distortion in the passband of the transmitter. However, the voltage ratio is decreased through mismatch in such a scheme by about a factor of Z_1/Z_2 (see eq. (15) of I, Sec. B) as against the case where the material of high acoustic impedance is used for the transmitting medium. For the case of a water line with mercury-backed crystals, $Z_1 = 1.5 \times 10^5$ and $Z_2 = 19.8 \times 10^5$ and the added loss amounts to about 22 db.

With the medium chosen, the range of the carrier frequency is clearly indicated. It must be chosen so that the transit attenuation will not be excessive but, if reflecting backings for the crystals, the attenuation must be large enough to absorb the accessory signals. Usually about 20 db one way is sufficient. One must bear in mind that a high carrier frequency means thinner crystals, more critical alignment and more severe mechanical tolerances. Lower carrier frequencies involve greater spreading of the beam and increased dispersion in velocity between the different frequency components of the signal pulse (I, Sec. C). In addition, where one wishes the pulse *envelope* to be faithfully reproduced regardless of phase, one requires a certain number of cycles per pulse. This involves a dependence of carrier frequency on the duration of the pulses to be delayed.

It has already been pointed out in I, Sec. B that to drive a crystal at an odd harmonic of its fundamental frequency generally increases the voltage loss and this procedure should only be employed for those frequencies at which a crystal operating on the fundamental would be too thin for manufacture or convenient handling.

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[3] G. W. Willard, *J. Acous. Soc. Am.*, 19, 1 (1947).
[4] C. E. Teete, Jr., *J. Acous. Soc. Am.*, 18, 488 (1946).

On the other hand, the tolerance on crystal thickness can be very loose if the crystal is operating into a medium of high acoustic impedance such as mercury. Crystals whose fundamental frequency lay outside the passband of the transmitter have been used satisfactorily in the mercury delay lines.

The size of the crystal diameter is primarily tied up with the problem of minimizing voltage loss through mismatch. It has been shown in I, Sec. B, that, for low impedance driving and receiving circuits, the minimum voltage loss for constant bandwidth occurs when the live capacity across the excited area of the crystal is of the same order as the stray capacity at the crystal when the latter has been reduced to a minimum. At present there is no known reason to have the inner cross section of the tube differ from that of the active area of the crystal. With smaller crystals and tubes, the beam spread is greater and the tube boundary conditions become more important. The effect of tubular attenuation is increased and also whatever pulse distortion may exist from velocity dispersion (I, Sec. C).

The number of channels and corner reflectors depends on the need for reducing the long dimension of the delay line. It is primarily a problem in packaging and utilization of space. The introduction of reflectors and additional channels complicates the manufacture and increases the need for accurate machining of individual pieces. It decreases, however, any difficulty that may be experienced with bending or sagging of the line.

The over-all mechanical tolerances are in effect determined when the medium and carrier frequency have been chosen. It is the ratio of wave length in the transmitting medium to crystal diameter which determines allowable tolerances. If $\sin \theta = 1.22 \lambda/d$, then something like 0.2θ is a permissible over-all tolerance. Where the final alignment of the crystal depends on the net result of several individual tolerances, such as is the case with many corner reflectors, the respective individual tolerances have to be proportionately more severe. For a corner reflector, for which the reflecting surfaces are accurately perpendicular, it is only necessary to align the reflector in the angle at right angles to the plane of the reflection.

D. AN EXAMPLE OF DELAY LINE DESIGN.

A line which was designed for 3,300 microseconds delay has been chosen as an example to illustrate some of the points discussed in the preceding section. The crystal end assemblies are shown in Figs. 6 and 7. Fig. 8 shows the corner reflector.

The line is variable in length with a variation of roughly 1 in. possible (about 17 microseconds at ordinary temperatures for mercury). This variation is made in the end assembly shown in Fig. 6, which was largely designed by Mr. P. C. Bettler. In this end assembly, the long

shaft holding the crystal, which protrudes into the end block, can be made to travel by means of a screw having 16 threads per inch. The mercury is retained by a packing nut and packing gland, consisting of thread or linen cloth impregnated with ceresine wax, a kind of paraffin.

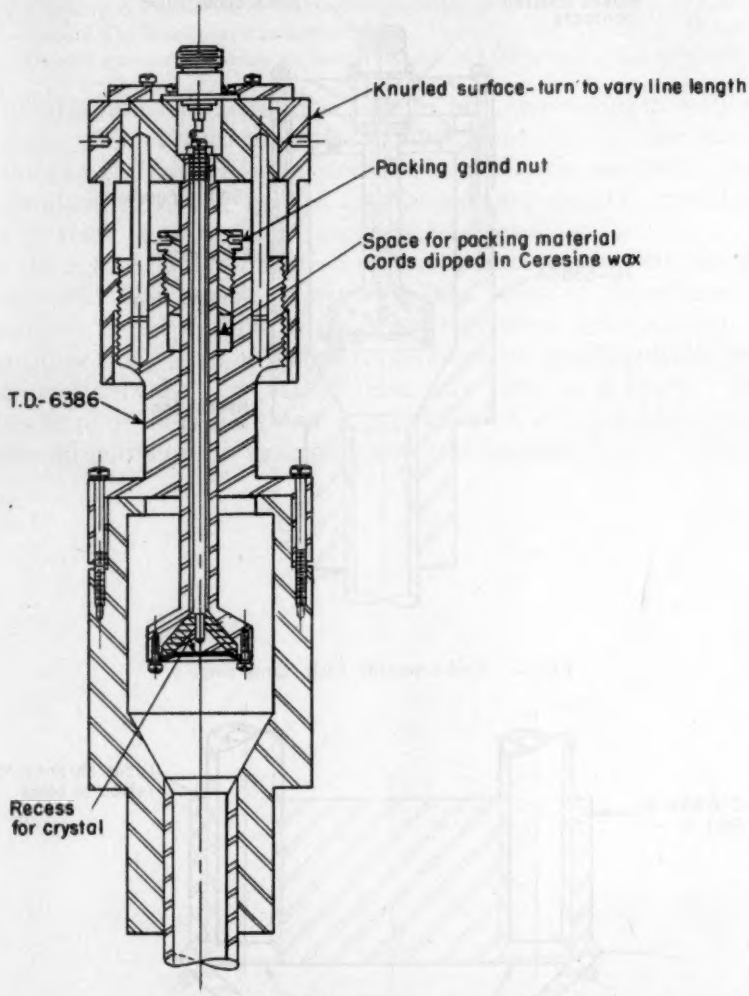

FIG. 6. Variable end assembly.

Ultrasonic 10 megacycle waves leaving the crystal of the adjustable crystal holder assembly travel to the reflector block, where they are twice reflected through 90°, and then continue back to the fixed crystal holder (Fig. 7), where they are picked up by a crystal similar to the transmitting crystal. These crystals are backed with a conical piece of stainless steel insulated by Bakelite from the rest of the crystal mount assembly.

The two crystal mount gaskets are made of polythene, which keeps the mercury from leaking. The long tubes are shrunk into the end blocks and reflector block. It is found that no mercury leaked when the

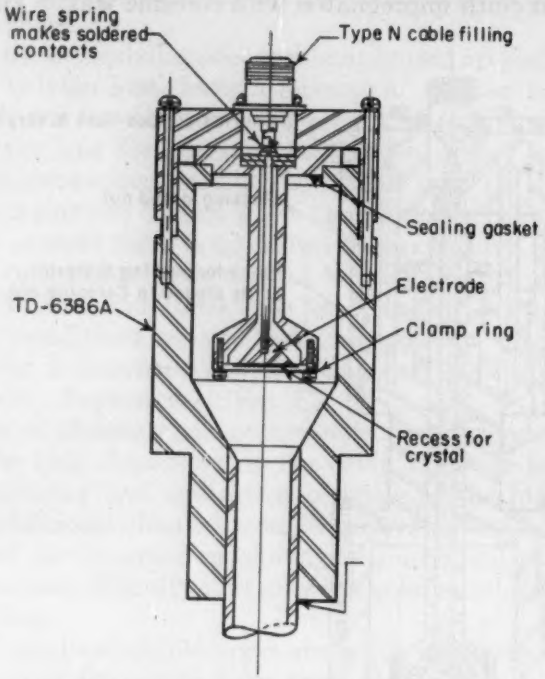

FIG. 7. End assembly with clamp ring.

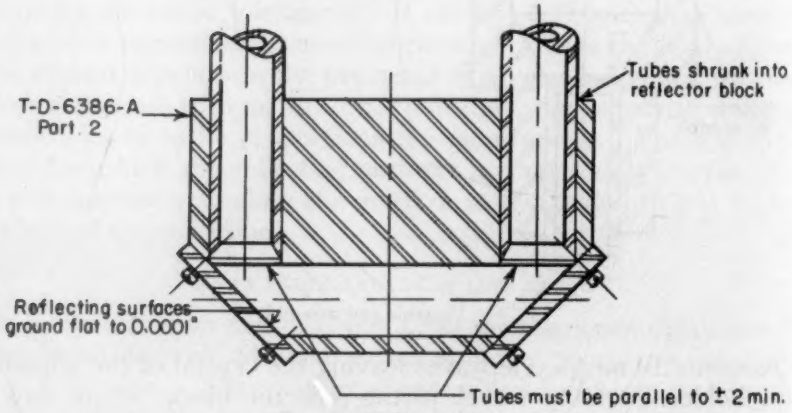
T-D-6386-A Part 2
Tubes shrunk into reflector block
Reflecting surfaces ground flat to 0.0001"
Tubes must be parallel to ± 2 min.

FIG. 8. Reflector block.

line is in a horizontal position, as it always is when operating. However, during filling, when the line was on end, small leaks were sometimes found at the shrunk fits mentioned above, since the mercury pressure is as high as 3 atmospheres at the reflector block (Fig. 8).

The end blocks are not completely filled with mercury; a volume of air of 3 cu. in. is left in each to allow for the expansion of the mercury should the temperature increase greatly in the field.

The constants for this delay line are as follows:

Capacity of adjustable crystal holder assembly	53 $\mu\mu$f
Capacity of (fixed) crystal holder	37 $\mu\mu$f
Overall attenuation (using an output resistor of 1,000 ohms)	60–70 db

The attenuation varies from line to line, depending mostly on the alignment, which is critical. The bandwidth of the line was determined by values of the capacities quoted above and their associated resistors. The bandwidth of the quartz crystal-mercury-quartz crystal system taken by itself is more than 6 megacycles broad.

In the actual field application four such lines are packed into parallel compartments of a special containing box. One of these lines is used to establish the repetition rate for the repeating pulse pattern. Two of the other lines were adjusted to give delays synchronizing with the repetition interval. The fourth line was used as a spare. Because all lines were in the same thermal environment, it was seldom necessary to make adjustments to maintain synchronization.

Three-Dimensional Inspection of Castings.—Inspection and layout of castings by a three-dimensional method of projecting accurate layout drawings upon a rough casting has been developed by G-E engineers at the Pittsfield Works and has seen several years of successful application to thousands of intricate ferrous and nonferrous castings. This method has effected considerable savings in time and material. Moreover, an unskilled operator with but a few days training can employ the method, whereas the usual bench layout requires inspectors familiar with machining practice.

Originally designed to inspect and lay out parts for subsequent machining, this method has also been used for a rapid inspection of finished parts. Projection may also be employed during an actual machining process, whether the part to be machined is stationary or revolving. The equipment assures layout within fifteen thousandths of an inch.

The installed apparatus consists of a layout image projector containing a photographic glass slide of the finished casting layout. The layout is projected by a lens directly upon the surface of the casting. To establish the plane of true projection and correct dimensions, a second projector—the inspection plane projector or light wand—over the inspection position is used. The light wand emits a sheet of light which falls vertically upon the casting.

Both projectors operate in unison through a selsyn system. Thus, the light wand always designates the proper plane for the main projector's inage as well as its correct focus and size.

The inspection pedestal assists in positioning the rough casting by means of an electric drive. An adjustable surface plate allows for the final location of the casting coincidental to the sheet of light from the inspection plane projector. Automatically, the finished layout will be projected properly on the rough casting.

When machining a piece which revolves and is symmetrical about the axis of rotation, the finished outline may be projected to serve as a template.

This template-projection system may be used in fabricating duplicate metal parts for tanks, boilers, and other burned-out and welded pieces. Variations of this system can be used for laying out intricate pieces, locating parts to be welded, shearing, and general layout operations.

To make this three-dimensional method practical, drawings of finished castings, which are suitable for photographing, are prepared. The size and focus of the drawing image on the ground glass of the camera are checked by microscope for accuracy.

In comparison to this three-dimensional inspection system, bench layout procedure required a highly skilled layout man. A reference surface was selected. Processing across or around the casting, each machinable area was checked for location and size with suitable tools, such as height gage, steel rule and scriber. Only by laying out the entire casting could the layout man be sure that all the machinable surfaces were properly located.

R. H. O.

AN INTEGRAL-EQUATION APPROACH TO PROBLEMS OF VIBRATING BEAMS.*

BY

WALTER T. WHITE, Sc.D.†

PART II.

5. CONSTRUCTION OF GREEN'S FUNCTION.

The normal modes of a vibrating beam depend on its *mass* and *stiffness*. For one-dimensional continuous beams, the mass and stiffness are each functions of a single variable along the length of the beam. The mass function is calculated from the physical specification of the beam while the stiffness function is specified by a *Green's function*. The Green's function gives the deflection at any point of the beam caused by application of a unit-concentrated load at any other point. The superposition principle is used with the differential equation of the beam for the construction of Green's functions.

For the freely vibrating beam, the partial differential equation is

$$\frac{\partial^2}{\partial z^2}\left(EI\frac{\partial^2 y}{\partial z^2}\right) = -m\frac{\partial^2 y}{\partial t^2}. \tag{37}$$

In this equation the mass m per unit length and the moment of inertia I are either constants or functions of z. Equation 37 is reduced to an ordinary differential equation by the substitution

$$y(z, t) = y(z)\cdot\sin \omega t. \tag{38}$$

By this substitution Eq. 37 becomes

$$\frac{d^2}{dz^2}\left(EI\frac{d^2 y}{dz^2}\right) = m\omega^2 y, \tag{39}$$

where y is a function of z alone and ω is the natural frequency of the beam in radians per second.

The term on the right of Eq. 39 represents the inertial loading of the beam and is replaced by a distributed loading $f(z)$ per unit length of beam. This substitution gives

$$\frac{d^2}{dz^2}\left(EI\frac{d^2 y}{dz^2}\right) = f(z). \tag{40}$$

* This paper is from a thesis, "Integral-Equation and Approximate Solutions for Normal Modes of Vibration," submitted to the department of Electrical Engineering, M.I.T., 1941.

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For a beam with uniform flexural rigidity, Eq. 40 becomes

$$EI\frac{d^4y}{dz^4} = f(z). \tag{41}$$

The solution of Eq. 40 or 41 is the *static* deflection of the beam caused by the loading $f(z)$.

The Green's function for a uniform beam is constructed by the solution of Eq. 41, with the appropriate boundary conditions, provided $f(z)$ is a *unit*-concentrated load. In the notation of operational calculus, a unit-concentrated load is written as $u_1(z - v)$, where the load is applied at $z = v$. With this substitution Eq. 41 becomes

$$EI\frac{d^4y}{dz^4} = u_1(z - v). \tag{42}$$

The boundary conditions for the cantilever are that the deflection and slope at the fixed end are zero, and the bending moment and shear at the free end are zero. These conditions are expressed by the relations

$$\left.\begin{aligned} y(0) = y'(0) &= 0 \\ EIy''(L) = EIy'''(L) &= 0 \end{aligned}\right\}. \tag{43}$$

The Laplace transformation [7] of Eq. 42 is

$$s^4Y(s) - y(0)s^3 - y'(0)s^2 - y''(0)s - y'''(0) = e^{-v}/EI, \tag{44}$$

where

$$\text{Laplace, } [y(z)] \equiv Y(s). \tag{45}$$

Division of Eq. 44 by s^4 and the inverse Laplace transformation gives

$$y(z) = y(0) + y'(0)z + y''(0)z^2/2! + y'''(0)z^3/3! + (z - v)^3u(z - v)/3!\,EI, \tag{46}$$

where $u(z - v)$ is the *unit function*, defined as zero for $z < v$ and unity for $z > v$. By application of the boundary conditions of Eq. 43, the constants $y''(0)$ and $y'''(0)$ are calculated. Evaluation of the constants gives the Green's function $G(z, v)$ for the cantilever as

$$\left.\begin{aligned} y(z) = G(z, v) &= z^2(3v - z)/6EI, \quad z \le v \\ &= v^2(3z - v)/6EI, \quad v \le z \end{aligned}\right\}. \tag{47}$$

This relation is identical with Eq. 2 which is the static deflection of a uniform cantilever caused by a unit-concentrated load applied at $z = v$.

By application of the appropriate boundary conditions, Green's functions for uniform beams with other types of end conditions are constructed from Eq. 46.

The construction of Green's functions for nonuniform beams requires the solution of two second-order differential equations. The

first of these equations gives the bending moment $M(z)$ for a beam as

$$M(z) = EI(z)\frac{d^2y}{dz^2}. \tag{48}$$

Substitution of this relation in Eq. 40 gives the second equation as

$$\frac{d^2M}{dz^2} = f(z). \tag{49}$$

A bending-moment Green's function is constructed from Eq. 48 when $M(z)$ is a *unit*-concentrated bending-moment; that is,

$$M(z) = u_{1m}(z - w) \tag{50}$$

is a bending moment that is unity at $z = w$ and zero elsewhere. Substitution in Eq. 48 gives

$$EI(z)\frac{d^2y}{dz^2} = u_{1m}(z - w). \tag{51}$$

Since $u_{1m}(z - w)$ has a non-zero value only at $z = w$ and $I(z)$ does not vanish along the beam, $I(z)$ is replaced by $I(w)$ and Eq. 51 written as

$$\frac{d^2y}{dz^2} = u_{1m}(z - w)/EI(w). \tag{52}$$

Solution of Eq. 52 by Laplace transformation gives

$$y(z) = y(0) + y'(0)z + (z - w)u(z - w)/EI(w). \tag{53}$$

Substitution of the boundary conditions for the cantilever gives the bending-moment Green's function $H(z, w)$ as

$$\left.\begin{aligned} y(z) = H(z, w) &= (z - w)/EI(w), \quad w \leq z \\ &= 0 \quad , \quad z \leq w \end{aligned}\right\}. \tag{54}$$

The solution of Eq. 49 when $f(z)$ is a unit-concentrated load applied at $z = v$ gives a bending-moment distribution $M(z, v)$ as

$$\left.\begin{aligned} M(z, v) &= (v - z), \quad z \leq v \\ &= 0 \quad , \quad v \leq z \end{aligned}\right\}. \tag{55}$$

To construct the Green's function, consider the deflection of the beam caused by an element of bending moment $M(z, v)\Delta z$ applied at $z = w$. For small values of Δz this is a concentrated bending moment and the deflection is

$$\Delta y = H(z, w)M(w, v)\Delta w, \tag{56}$$

where $H(z, w)$ is given by Eq. 54. The deflection caused by the bending-moment distribution of Eq. 55 is the required Green's function and by

integration of Eq. 56 is given as

$$G(z, v) = \int_0^L H(z, w)M(w, v)dw. \tag{57}$$

By substitution from Eqs. 54 and 55 in Eq. 57, the Green's function for the non-uniform cantilever is

$$\left.\begin{aligned} G(z, v) &= \int_0^z \frac{(z - w)(v - w)dw}{EI(w)}, \qquad z \leq v \\ &= \int_0^v \frac{(z - w)(v - w)dw}{EI(w)}, \qquad v \leq z \end{aligned}\right\}. \tag{57a}$$

When the function $I(w)$ is known, these integrals can be evaluated by any of the usual methods.

By application of the superposition principle and the appropriate boundry conditions, the Green's functions for nonuniform beams with other types of end conditions are found by methods analogous to the foregoing.

The Green's functions for naturally twisted beams are constructed by consideration of the coupling between the axes of maximum and minimum flexural rigidity. In this discussion a naturally twisted beam in an unstrained condition has: (a) the ellipses of inertia of all sections so displaced that they remain parallel; and (b) the centers of gravity of all sections along a straight line.

The coordinate system of Fig. 2 is used for the naturally twisted beam. Coordinate z extends along the length of the beam. Coordinates x and y are respectively the axes of maximum and minimum inertiae at the section $z = 0$. Coordinates α and β are respectively the axes of maximum and minimum inertiae at sections along the beam. The angle θ is the rotation of the α and β axes. The positive directions for the bending-moment vectors M_x, M_y, M_α, and M_β are shown in Fig. 2b. The principal moments of inertia are I_α and I_β ($I_\alpha < I_\beta$).

The differential equations for a freely vibrating, naturally twisted beam as reduced from the partial differential equations are

$$\left.\begin{aligned} d^2M_x/dz^2 &= m\omega^2x(z) \\ d^2M_y/dz^2 &= m\omega^2y(z) \end{aligned}\right\}. \tag{58}$$

The terms on the right of Eqs. 58 result from the inertial loadings and are replaceable respectively by static-load functions $f_x(z)$ and $f_y(z)$.

The bending-moment distribution caused by unit-concentrated loads $u_{1x}(z - v)$ and $u_{1y}(z - v)$ applied respectively in the x and y directions at $z = v$ is obtained by solutions of the differential equations

$$\left.\begin{aligned} d^2M_x/dz^2 &= u_{1x}(z - v) \\ d^2M_y/dz^2 &= u_{1y}(z - v) \end{aligned}\right\}. \tag{59}$$

For the cantilever, the solutions of Eqs. 59 are

$$\left.\begin{aligned} M_x(z, v) &= (v - z)u(v - z) \\ M_y(z, v) &= (v - z)u(v - z) \end{aligned}\right\}. \tag{60}$$

The bending-moment Green's functions are obtained from the differential equations for the deflection of the beam in the α and β coordinates.

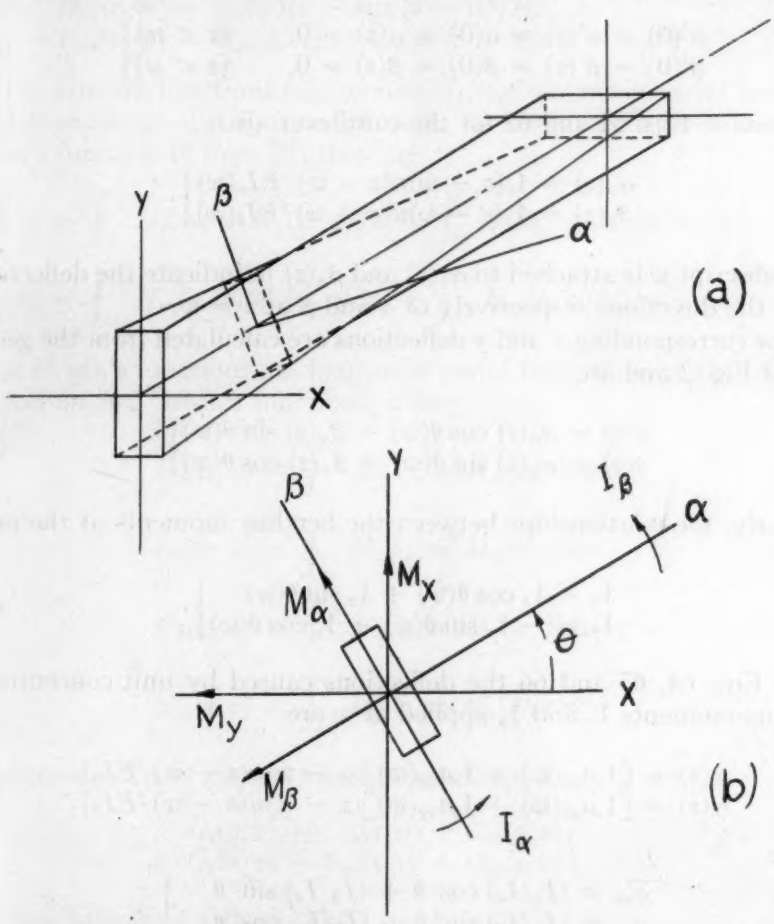

FIG. 2. Naturally twisted beam.

Since these equations are obtained by consideration of the curvature of a differential element, they are not affected by the twist of the beam. The equations for the deflections α and β are

$$\left.\begin{aligned} d^2\alpha/dz^2 &= M_\alpha/EI_\alpha \\ d^2\beta/dz^2 &= M_\beta/EI_\beta \end{aligned}\right\}. \tag{61}$$

These equations are solved for unit-concentrated bending-moments

$$\left.\begin{aligned} M_\alpha &= 1_\alpha u_{1m}(z-w) \\ M_\beta &= 1_\beta u_{1m}(z-w) \end{aligned}\right\}, \tag{62}$$

applied at $z = w$. The boundary conditions for a cantilever with the bending moments of Eqs. 62 are

$$\left.\begin{aligned} \alpha'(0) = \alpha'(z) = \alpha(0) = \alpha(z) = 0, \qquad (z < w) \\ \beta'(0) = \beta'(z) = \beta(0) = \beta(z) = 0, \qquad (z < w) \end{aligned}\right\}. \tag{63}$$

Solutions of Eqs. 61 and 62 for the cantilever are

$$\left.\begin{aligned} \alpha_w(z) &= 1_\alpha(z-w)u(z-w)/EI_\alpha(w) \\ \beta_w(z) &= 1_\beta(z-w)u(z-w)/EI_\beta(w) \end{aligned}\right\}. \tag{64}$$

The subscript w is attached to $\alpha_w(z)$ and $\beta_w(z)$ to indicate the deflections are in the directions respectively of α and β at $z = w$.

The corresponding x and y deflections are calculated from the geometry of Fig. 2 and are

$$\left.\begin{aligned} x(z) &= \alpha_w(z)\cos\theta(w) - \beta_w(z)\sin\theta(w) \\ y(z) &= \alpha_w(z)\sin\theta(w) + \beta_w(z)\cos\theta(w) \end{aligned}\right\}. \tag{65}$$

Similarly, the relationships between the bending moments at the point w are

$$\left.\begin{aligned} 1_\alpha &= 1_x\cos\theta(w) + 1_y\sin\theta(w) \\ 1_\beta &= -1_x\sin\theta(w) + 1_y\cos\theta(w) \end{aligned}\right\}. \tag{66}$$

From Eqs. 64, 65 and 66 the deflections caused by unit-concentrated bending-moments 1_x and 1_y applied at w are

$$\left.\begin{aligned} x(z) &= [1_x a_{xx}(w) + 1_y a_{xy}(w)](z-w)u(z-w)/EI_0 \\ y(z) &= [1_x a_{yx}(w) + 1_y a_{yy}(w)](z-w)u(z-w)/EI_0 \end{aligned}\right\}, \tag{67}$$

where

$$\left.\begin{aligned} a_{xx} &\equiv (I_0/I_\alpha)\cos^2\theta + (I_0/I_\beta)\sin^2\theta \\ a_{yy} &\equiv (I_0/I_\alpha)\sin^2\theta + (I_0/I_\beta)\cos^2\theta \\ a_{xy} &= a_{yx} \equiv (I_0/I_\alpha - I_0/I_\beta)\sin\theta\cos\theta \end{aligned}\right\}, \tag{68}$$

and

$$I_0 \equiv I_\alpha(0). \tag{69}$$

The deflections $x(z)$ and $y(z)$ in Eqs. 67 are the bending-moment Green's functions $H_x(z, w)$ and $H_y(z, w)$, since they are deflections caused by unit-concentrated bending-moments. Equations 67 are written in

terms of *self* and *mutual* bending-moment Green's functions to give

$$\left.\begin{aligned} H_x(z, w) &= 1_x H_{xx}(z, w) + 1_y H_{xy}(z, w) \\ H_y(z, w) &= 1_x H_{yx}(z, w) + 1_y H_{yy}(z, w) \end{aligned}\right\}, \tag{70}$$

where the *self* and *mutual* bending-moment Green's functions are

$$\left.\begin{aligned} H_{xx}(z, w) &\equiv a_{xx}(w)(z - w)u(z - w)/EI_0 \\ H_{yy}(z, w) &\equiv a_{yy}(w)(z - w)u(z - w)/EI_0 \\ H_{xy}(z, w) &= H_{yx}(z, w) \equiv a_{xy}(w)(z - w)u(z - w)/EI_0 \end{aligned}\right\}. \tag{71}$$

The Green's functions $G_x(z, v)$ and $G_y(z, v)$ are constructed from the bending-moment distribution of Eqs. 60 and the bending-moment Green's function of Eqs. 70; they are

$$\left.\begin{aligned} G_x(z, v) &= \int_0^L H_{xx}(z, w) M_x(w, v)dw + \int_0^L H_{xy}(z, w)\, M_y(w, v)dw \\ G_y(z, v) &= \int_0^L H_{yx}(z, w) M_x(w, v)dw + \int_0^L H_{yy}(z, w) M_y(w, v)dw \end{aligned}\right\}. \tag{72}$$

These Green's functions are in a more useful form when separated into *self* and *mutual* Green's functions, where

$$\left.\begin{aligned} G_{xx}(z, v) &\equiv \int_0^L H_{xx}(z, w) M_x(w, v)dw \\ G_{xy}(z, v) &\equiv \int_0^L H_{xy}(z, w) M_y(w, v)dw \\ G_{yx}(z, v) &\equiv \int_0^L H_{yx}(z, w) M_x(w, v)dw \\ G_{yy}(z, v) &\equiv \int_0^L H_{yy}(z, w) M_y(w, v)dw \end{aligned}\right\}. \tag{73}$$

In terms of self and mutual Green's functions, Eqs. 72 are

$$\left.\begin{aligned} G_x(z, v) &= G_{xx}(z, v) + G_{xy}(z, v) \\ G_y(z, v) &= G_{yx}(z, v) + G_{yy}(z, v) \end{aligned}\right\}. \tag{72a}$$

From Eqs. 60 it follows that

$$M_x(w, v) = M_y(w, v), \tag{60a}$$

and therefore in Eqs. 72a

$$G_{xy}(z, v) = G_{yx}(z, v). \tag{74}$$

The formulation of the integral equations follows from integration of the Green's functions and the inertial loadings of Eqs. 58. Accord-

ingly, the integral equations are

$$\left.\begin{aligned} x(z) &= \omega^2 \int_0^L G_{xx}(z, v)m(v)x(v)dv + \omega^2 \int_0^L G_{xy}(z, v)m(v)y(v)dv \\ y(z) &= \omega^2 \int_0^L G_{yx}(z, v)m(v)x(v)dv + \omega^2 \int_0^L G_{yy}(z, v)m(v)y(v)dv \end{aligned}\right\}. \quad (75)$$

The kernels of these integral equations are placed in a symmetric form by transformations similar to Eqs. 16 and 17. Hence, if

$$\left.\begin{aligned} X(z) &\equiv x(z)\sqrt{m(z)} \\ Y(z) &\equiv y(z)\sqrt{m(z)} \\ K_{xx}(z, v) &\equiv G_{xx}(z, v)\sqrt{m(z)m(v)} \\ K_{xy}(z, v) &\equiv G_{xy}(z, v)\sqrt{m(z)m(v)} \\ K_{yx}(z, v) &\equiv G_{yx}(z, v)\sqrt{m(z)m(v)} \\ K_{yy}(z, v) &\equiv G_{yy}(z, v)\sqrt{m(z)m(v)} \\ \lambda &\equiv \omega^2 \end{aligned}\right\}, \quad (76)$$

the integral equations are

$$\left.\begin{aligned} X(z) &= \lambda \int_0^L K_{xx}(z, v)X(v)dv + \lambda \int_0^L K_{xy}(z, v)\,Y(v)dv \\ Y(z) &= \lambda \int_0^L K_{yx}(z, v)X(v)dv + \lambda \int_0^L K_{yy}(z, v)\,Y(v)dv \end{aligned}\right\}. \quad (77)$$

From Eqs. 74 and 76 it follows that

$$K_{xy}(z, v) = K_{yx}(z, v). \quad (78)$$

The orthogonality condition for the pair of integral equations in Eqs. 77 is obtained by consideration of the separate eigenvalues λ_i and λ_j, and the corresponding eigenfunctions $X_i(z)$, $X_j(z)$, $Y_i(z)$ and $Y_j(z)$ of the symmetric kernels. The eigenfunctions are solutions of Eqs. 77; hence

$$\left.\begin{aligned} X_i(z) &= \lambda_i \int_0^L K_{xx}(z, v)X_i(v)dv + \lambda_i \int_0^L K_{xy}(z, v)\,Y_i(v)dv \\ Y_i(z) &= \lambda_i \int_0^L K_{yx}(z, v)X_i(v)dv + \lambda_i \int_0^L K_{yy}(z, v)\,Y_i(v)dv \end{aligned}\right\}, \quad (79)$$

and

$$\left.\begin{aligned} X_j(z) &= \lambda_j \int_0^L K_{xx}(z, v)X_j(v)dv + \lambda_j \int_0^L K_{xy}(z, v)\,Y_j(v)dv \\ Y_j(z) &= \lambda_j \int_0^L K_{yx}(z, v)X_j(v)dv + \lambda_j \int_0^L K_{yy}(z, v)\,Y_j(v)dv \end{aligned}\right\}. \quad (80)$$

Multiply Eqs. 79 and 80 respectively by $\lambda_j X_j(z)$, $\lambda_j Y_j(z)$, $\lambda_i X_i(z)$ and $\lambda_i Y_i(z)$ and by subtraction form the following equations:

$$\left.\begin{aligned}(\lambda_j - \lambda_i)X_i(z)X_j(z) &= \lambda_i\lambda_j\Big[\int_0^L K_{xx}(z, v)X_j(z)X_i(v)dv \\ &\quad + \int_0^L K_{xy}(z, v)X_j(z)Y_i(v)dv \\ &\quad - \int_0^L K_{xx}(z, v)X_i(z)X_j(v)dv \\ &\quad - \int_0^L K_{xy}(z, v)X_i(z)Y_j(v)dv\Big] \\ (\lambda_j - \lambda_i)Y_i(z)Y_j(z) &= \lambda_i\lambda_j\Big[\int_0^L K_{yx}(z, v)Y_j(z)X_i(v)dv \\ &\quad + \int_0^L K_{yy}(z, v)Y_j(z)Y_i(v)dv \\ &\quad - \int_0^L K_{yx}(z, v)Y_i(z)X_j(v)dv \\ &\quad - \int_0^L K_{yy}(z, v)Y_i(z)Y_j(v)dv\Big]\end{aligned}\right\}. \tag{81}$$

The orthogonality condition is given by integration of Eqs. 81 with respect to z, and the addition of the integrated Eqs. 81 after the appropriate interchange of z and v; the result is

$$(\lambda_j - \lambda_i)\int_0^L [X_i(z)X_j(z) + Y_i(z)Y_j(z)]dz = 0. \tag{82}$$

Since the eigenvalues λ_i and λ_j are postulated different, the orthogonality condition is obtained from Eq. 82 as

$$\int_0^L [X_i(z)X_j(z) + Y_i(z)Y_j(z)]dz = 0, \qquad i \neq j. \tag{83}$$

Substitution from Eqs. 76 gives the orthogonality condition in terms of $x(z)$ and $y(z)$ as

$$\int_0^L [x_i(z)x_j(z) + y_i(z)y_j(z)]m(z)dz = 0, \qquad i \neq j. \tag{84}$$

If $j = i$ the normalization constant N_i is given by the relation

$$N_i = \int_0^L [x^2{}_i(z) + y^2{}_i(z)]m(z)dz. \tag{85}$$

The bilinear expansions for the self and mutual kernels can be written as

$$\left.\begin{aligned} K_{xx}(z, v) &= \sum_1^\infty \frac{X_n(z)X_n(v)}{\lambda_n N_n} \\ K_{yy}(z, v) &= \sum_1^\infty \frac{Y_n(z)Y_n(v)}{\lambda_n N_n} \\ K_{xy}(z, v) &= \sum_1^\infty \frac{X_n(z)Y_n(v)}{\lambda_n N_n} \\ K_{yx}(z, v) &= \sum_1^\infty \frac{Y_n(z)X_n(v)}{\lambda_n N_n} \end{aligned}\right\}. \tag{86}$$

The correctness of these relations can be demonstrated by substitution in Eqs. 77 for the nth eigenfunction. The uniform convergence is assured for practical problems.

Substitutions from Eqs. 76 in Eqs. 86 give the expansions for the self and mutual Green's functions as

$$\left.\begin{aligned} G_{xx}(z, v) &= \sum_1^\infty \frac{x_n(z)x_n(v)}{\lambda_n N_n} \\ G_{yy}(z, v) &= \sum_1^\infty \frac{y_n(z)y_n(v)}{\lambda_n N_n} \\ G_{xy}(z, v) &= \sum_1^\infty \frac{x_n(z)y_n(v)}{\lambda_n N_n} \\ G_{yx}(z, v) &= \sum_1^\infty \frac{y_n(z)x_n(v)}{\lambda_n N_n} \end{aligned}\right\}. \tag{87}$$

6. NORMAL MODES OF A TWISTED TURBINE BLADE.

Application of the integral-equation method is made to the calculation of the normal modes of a twisted turbine blade. This example is restricted to a single stationary blade although the method can be extended to a single rotating blade or to a group of rotating blades with shroud bands.

It is convenient to non-dimensionalize the integral equation for the twisted beam by the substitutions in Eqs. 75 of

$$\left.\begin{aligned} w &= w'L \\ v &= v'L \\ z &= z'L \end{aligned}\right\}. \tag{88}$$

The non-dimensional equations are written with primes omitted as

$$\left.\begin{aligned} x(z) &= \frac{\lambda}{m_0}\int_0^1 k_{xx}(z, v)m(v)x(v)dv + \frac{\lambda}{m_0}\int_0^1 k_{xy}(z, v)m(v)y(v)dv \\ y(z) &= \frac{\lambda}{m_0}\int_0^1 k_{xy}(z, v)m(v)x(v)dv + \frac{\lambda}{m_0}\int_0^1 k_{yy}(z, v)m(v)y(v)dv \end{aligned}\right\}, \tag{89}$$

where

$$\left.\begin{aligned} \lambda &= m_0L^4\omega^2/EI_0 \\ k_{xx}(z, v) &= EI_0G_{xx}(z, v)/L^3 \\ k_{yy}(z, v) &= EI_0G_{yy}(z, v)/L^3 \\ k_{xy}(z, v) &= EI_0G_{xy}(z, v)/L^3 \end{aligned}\right\}. \tag{90}$$

The kernels of Eqs. 89 are calculated from Eqs. 60, 71, 73 and 90 as

$$\left.\begin{aligned} k_{xx}(z, v) &= \int_0^z a_{xx}(w)(z - w)(v - w)dw \\ k_{yy}(z, v) &= \int_0^z a_{yy}(w)(z - w)(v - w)dw \\ k_{xy}(z, v) &= \int_0^z a_{xy}(w)(z - w)(v - w)dw \end{aligned}\right\}, \qquad z \leq v. \tag{91}$$

The kernels are symmetric in z and v, and therefore

$$k(z, v) = k(v, z). \tag{92}$$

TABLE III.
Data for Turbine Blade.

z	$\frac{m(z)}{m_0}$	$I_\alpha(z)$	$I_\beta(z)$	$\theta(z)$	$a_{xx}(z)$	$a_{yy}(z)$	$a_{xy}(z)$
0.0	1.000	0.143	0.818	19.0°	0.913	0.263	0.253
0.1	0.941	0.127	0.816	24.2	0.965	0.335	0.355
0.2	0.876	0.103	0.810	29.3	1.098	0.470	0.518
0.3	0.808	0.081	0.784	34.0	1.272	0.680	0.736
0.4	0.725	0.062	0.732	38.5	1.491	1.013	1.025
0.5	0.646	0.049	0.712	42.4	1.671	1.425	1.339
0.6	0.588	0.039	0.730	45.6	1.895	1.966	1.725
0.7	0.552	0.030	0.772	48.6	2.194	2.761	2.270
0.8	0.536	0.022	0.818	51.3	2.676	4.049	3.120
0.9	0.532	0.016	0.866	53.8	3.337	6.058	4.320
1.0	0.548	0.010	0.914	56.1°	4.338	9.429	6.220

L = 31.6 cm.; length of beam.
A_0 = 1.53 cm.²; cross-sectional area at $z = 0$.
I_0 = 0.143 cm.⁴; moment of inertia $I_\alpha(0)$.
m_0 = 1.227×10^{-6} kg.-sec.²/cm.²; mass per unit length at $z = 0$.
E = 2.05×10^6 kg./cm.²; Young's modulus.
p = 0.00785 kg./cm.³; density of material.
$\sqrt{\frac{EI_0}{m_0L^4}} = \frac{\omega}{\sqrt{\lambda}}$ = 155 radians/sec.; ratio natural frequency to square root of eigenvalue.
units: cm.-kg.-sec.; length-force-time.

Data for a typical turbine blade are given in Table III. The coordinate system of Fig. 2 applies to these data. The coefficients a_{zz}, a_{yy} and a_{zy} are calculated from Eqs. 68.

The self and mutual kernels are calculated from Eqs. 91 by the cinema integraph.[5] The results are in Table IV.

TABLE IV.

Self and Mutual Kernels of Twisted Turbine Blade.

(Multiply values by 10^{-4})

$k_{zz}(s, v)$

s / v	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0										
0.1	0	3									
0.2	0	7	26								
0.3	0	12	45	89							
0.4	0	16	64	133	210						
0.5	0	21	84	178	293	428					
0.6	0	26	103	223	375	564	769				
0.7	0	30	122	268	458	699	973	1265			
0.8	0	35	141	311	540	835	1179	1560	1956		
0.9	0	39	160	356	622	971	1384	1854	2361	2909	
1.0	0	44	180	401	705	1106	1590	2149	2767	3452	4166

$k_{yy}(s, v)$

s / v	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0										
0.1	0	0									
0.2	0	2	10								
0.3	0	3	16	27							
0.4	0	4	23	43	76						
0.5	0	6	30	59	109	164					
0.6	0	7	37	75	142	225	321				
0.7	0	8	44	91	174	284	424	592			
0.8	0	9	50	106	206	344	525	754	1001		
0.9	0	11	57	122	238	403	626	916	1252	1612	
1.0	0	12	64	138	271	463	727	1078	1504	1991	2543

$k_{zy}(s, v)$

s / v	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0										
0.1	0	2									
0.2	0	3	13								
0.3	0	4	19	33							
0.4	0	6	25	49	84						
0.5	0	8	32	66	118	176					
0.6	0	9	39	84	152	237	342				
0.7	0	10	45	101	186	298	446	597			
0.8	0	12	51	117	219	361	549	758	996		
0.9	0	15	58	133	253	423	652	920	1238	1587	
1.0	0	16	64	150	287	484	756	1082	1482	1943	2440

Generating functions are selected as

$$\left.\begin{aligned} g_{x0}(v) &= k_{xx}(0.7, v) \\ g_{y0}(v) &= k_{yy}(0.7, v) \end{aligned}\right\}. \tag{93}$$

Iteration of these generating functions in Eqs. 89 gives

$$\left.\begin{aligned} g_{x1} &= \left(s\int_0^1 k_{xx} m g_{x0} dv + \int_0^1 k_{xy} m g_{y0} dv\right) \Big/ m_0 \\ g_{y1} &= \left(s\int_0^1 k_{xy} m g_{x0} dv + \int_0^1 k_{yy} m g_{y0} dv\right) \Big/ m_0 \end{aligned}\right\}. \tag{94}$$

The constant s has been introduced as a scale factor, since the relative magnitudes of the generating functions g_{x0} and g_{y0} are arbitrarily established in Eqs. 93.

The appropriate value for s is found by making each of Eqs. 94 yield the same value for λ. For calculations of λ define B, C, D, E, F, and J as follows:

$$B = \int_0^1 g_{x0}(z)dz,$$

$$E = \int_0^1 g_{y0}(z)dz,$$

$$Cs + D = \int_0^1 g_{x1}(z)dz,$$

$$Fs + J = \int_0^1 g_{y1}(z)dz.$$

Hence for each iteration λ is given by

$$\lambda = \frac{Bs}{Cs + D} = \frac{E}{Fs + J}.$$

From this relation the following quadratic is obtained for calculations of s:

$$s^2 + (BJ - CE)s/BF - DE/BF = 0.$$

The positive root of s corresponds to the first mode and the negative root to the second mode.

Successive iteration and calculation of scale factors as in the foregoing leads to the eigenvalues and eigenfunctions for the first and second modes. The approximate eigenvalues are given by the first iteration, but additional iterations are required for the eigenfunctions to become stabilized. The resulting eigenvalues and eigenfunctions for the turbine blade are given in Table V. It is of interest to note that the eigen-

functions satisfy the orthogonality relation of Eqs. 84, although this relationship was not used in the foregoing iterations.

TABLE V.

Normal Modes of Twisted Turbine Blade.

z	First Mode		Second Mode	
	$x_1(z)$	$y_1(z)$	$x_2(z)$	$y_2(z)$
0.0	0.000	0.000	0.000	0.000
0.1	0.012	0.005	−0.018	0.001
0.2	0.047	0.023	−0.070	0.021
0.3	0.104	0.050	−0.146	0.040
0.4	0.183	0.095	−0.217	0.090
0.5	0.284	0.155	−0.289	0.164
0.6	0.408	0.234	−0.330	0.266
0.7	0.544	0.329	−0.362	0.423
0.8	0.690	0.432	−0.358	0.593
0.9	0.843	0.545	−0.340	0.787
1.0	1.000	0.661	−0.312	0.996
	$\lambda_1 = 14.93.$		$\lambda_2 = 98.7.$	
	$\omega_1 = 596$ radians/sec.		$\omega_2 = 1541$ radians/sec.	
	$f_1 = 95$ cycles/sec.		$f_2 = 245$ cycles/sec.	

Experimental observations of the first two natural frequencies for the turbine blade give the following:

First mode: 93 to 102 cycles/sec.
Second mode: 225 to 253 cycles/sec.

The calculated values in Table V check these observations and were made without prior knowledge of the experimental values.

7. CONCLUSIONS.

Integral equations are highly effective for the solution of problems in vibrations. Because of the closeness of the integral equation formulation to the physical picture, much simplification results when problems are studied directly with integral equations rather than by differential equations. The variations of physical properties or discontinuities cause no particular difficulty when they appear under the integral sign. Boundary conditions are contained in the Green's function and are handled there when the beam is subjected to the elementary unit-concentrated force. Once taken care of in the construction of the Green's function, the boundary conditions require no further consideration.

Much emphasis is placed on practical methods of solving the integral equation. If a mathematical machine, such as the differential analyzer, is available for evaluation of the integrals, the solution by successive approximations is easily obtained. On the other hand, because of the

rapid convergence of the successive approximations, an ordinary calculating machine is an economical tool.

When modes higher than the fundamental are required, the method of successive approximations must be used with care, or the solution will converge to the fundamental mode. Perhaps the application of the so-called *perturbated kernel* is the preferred method of solution, particularly if the integrals are evaluated only approximately. When the perturbated kernel is used, the importance of accurate physical data is immediately apparent. Since the kernel contains much of the physical data for the problem, it generally is found that the perturbated kernel is controlled largely by the tolerances on the physical data. This difficulty indicates no defect in the method of solution, but instead it boldly shows the engineer the accuracy of the solutions for the higher modes, rather than masking the accuracy under an elaborate calculation procedure.

The twisted turbine blade is an interesting problem to show the power of the integral-equation method in dealing with continuous systems where coupling exists. The agreement between the calculated natural frequencies and the experimental values probably is better than generally can be expected for so complex a problem. Yet where a large disagreement is found, the difficulty probably is caused by inaccurate physical data. The construction of the Green's function for the turbine blade shows the value of the bending-moment Green's function as an aid for construction of the final Green's function. The derivation of the orthogonality condition for the turbine blade is a special case of a development that may be required when higher modes are to be calculated for very complex systems. Of course, no orthogonality condition is required if only the fundamental mode is to be calculated.

In conclusion, the integral-equation approach to vibration problems is of particular value when there are variations of the physical properties of the vibrating member. As applied to beam vibration, the integral-equation method applies when simple bending theory is used, or when consideration is given to effects of rotary inertia, shearing force, longitudinal inertia, gyroscopic moment, etc. The technique is to consider the influence of each factor on the beam deflection when that factor is *acting alone*, and then by superposition to add all the individual influences to form the integral equation of the vibrating beam. The resultant equation generally is solved most directly by the method of successive approximations.

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Experimental Gas Turbines Being Constructed.—Two experimental gas turbines, one a 5000-kw. machine for electric power stations, and the other a 4800-hp. unit, are being constructed by the General Electric Company.

Alan Howard, G-E turbine-generator design engineer at Schenectady who headed design and development of the TG-100 Propjet, and TG-180 jet engines now powering the latest U. S. fighter planes, revealed that the company is also concentrating its facilities on gas turbine development in the nonaircraft field.

Shop tests of the 4800-hp. turbine are scheduled to begin soon. The stationary power plant is in the design stage and will not undergo factory test until later.

The 4800-hp. gas turbine will burn Bunker "C" fuel, although work is being carried on in the use of pulverized coal in order to permit more diversified application of gas turbines. The 4800-hp. unit being constructed is approximately 19 feet long and weighs between 2½ to 4 pounds per horsepower.

"We are making drawings," Howard said, "for a 5000-kw. plant suitable for central station or industrial applications. We believe this type of plant, when developed, will prove to have very competitive all-around econony and characteristics when compared with other types of plants in similar ratings."

In this stationary gas turbine, which also will be powered with oil, air flows through two compressors, an intervening intercooler, a regenerator and then into six combustion chambers. Nominal inlet temperature to the turbine is 1500° F., with the gases expanding first through a two-stage turbine which drives the high-pressure compressor. Turbine and compressor run at a constant speed of approximately 8700 r.p.m.

The rest of the expansion occurs through the single-stage low-pressure turbine which drives the low-pressure compressor.

R. H. O.

Trend of Power Transmission Economics Toward Operation of Large Generating Stations at High Circuit Loadings.—Power transmission systems are now entering a stage in their development where added generating capacity will be located at relatively greater distances from the utilization areas, S. B. Crary and I. B. Johnson of General Electric's analytical Division told A.I.E.E. members at recent meetings in Montreal and Quebec.

"Economics of power transmission will be toward development of large generating stations for connection to well integrated-systems so that transmission may be accomplished at high circuit loading and load factors," the G-E engineers said.

The two men indicated that operations of high-voltage systems with grounded neutrals materially reduces transmission costs by allowing for the use of the next lower level of transformer insulation as well as reducing transmission line expenses.

According to Crary and Johnson, the 360-kv. voltage level is about the most economical voltage for distances in the range from 300 to 600 miles when line compensation is used for power transfers of 300,000 to 350,000 kw. per circuit.

"230 to 287 kv. levels," they said, "appear to be most economical for heavily loaded long-distance transmission line conditions such as exist in this country today."

R. H. O.

PARTIALLY PLASTIC THICK-WALLED TUBES.[1]

BY

C. W. MacGREGOR,[2] L. F. COFFIN, JR.,[3] and J. C. FISHER.[4]

ABSTRACT.

A theory is presented for the partial plastic yielding of thick-walled cylindrical tubes acted upon by any combination of internal pressure, external pressure and end load when the material follows an arbitrary stress-strain law. The solution combines (a) the distortion energy theory of plastic flow and (b) the effects of elastic compressibility of the plastic material.

Numerical values for stresses and strains are given for certain special cases of internal pressure in thick-walled open-ended tubes for an idealized stress-strain law, and the results are compared with earlier approximate theories.

The maximum internal pressure which can be withstood by a thick-walled tube of a ductile metal considerably exceeds the pressure at which plastic flow begins. As the pressure increases from an initial value of zero, the entire tube is at first elastic, and the stresses at any location in the tube are given by the well-known expressions [5]

$$\left.\begin{aligned} \sigma_r &= -\frac{\left(\frac{R_1}{r}\right)^2 - 1}{\left(\frac{R_1}{R_0}\right)^2 - 1} p_0, \\ \sigma_t &= \frac{\left(\frac{R_1}{r}\right)^2 + 1}{\left(\frac{R_1}{R_0}\right)^2 - 1} p_0, \\ \sigma_z &= \frac{F}{\pi(R_1^2 - R_0^2)}. \end{aligned}\right\} \tag{1}$$

[1] This paper reports a portion of an investigation conducted for the Bureau of Ordnance, United States Navy. The work was done in the Research Laboratories for Mechanics of Materials, Mechanical Engineering Department, at the Massachusetts Institute of Technology. This material has been released for publication by the Chief of the Bureau of Ordnance. The opinions expressed herein are those of the authors and should be in no way interpreted as those of the Navy Department.

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[5] See Nomenclature for definition of symbols.

These equations are valid until the pressure reaches a value at which the metal at the bore of the tube becomes plastic (1).[6]

As the pressure increases above this critical value, a zone of yielding moves outward from the bore through the tube. More specifically, there is a cylindrical bounding surface called the plastic-elastic boundary which divides the inner and plastically deformed material from the

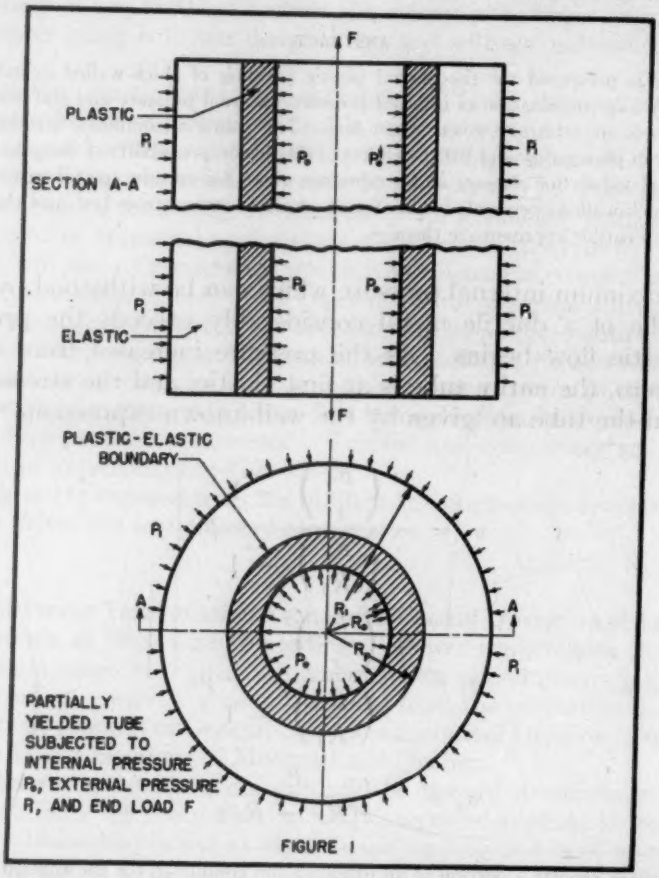


FIGURE 1

outer and still elastic material. At this stage the tube is said to be partially yielded. As the pressure is increased, the plastic-elastic boundary continues outward until it coincides with the outer surface, and the tube is then said to be completely yielded. A tube subjected to external pressure as well as internal pressure and end load will yield when the difference between the internal and external pressures is large enough. Such a tube, partially yielded, is shown schematically in Fig. 1.

[6] Numbers in parentheses refer to Bibliography at end of paper.

The fact that the internal pressure in a thick-walled tube can rise considerably above the value for initial yielding before the tube becomes completely plastic is of practical interest. With regard to high pressure tubing and other high pressure equipment, it assures a factor of safety considerably greater than that based only upon initial yielding. For this reason it is of interest to know the ratio of pressures for complete to initial yielding for a tube of a given material. In some instances, as for example in gun design, it may prove desirable to provide for operation at pressures exceeding that for initial yielding. An accurate knowledge of the depth of yielding, the factor of safety and the expected deformations is of considerable importance.

If a tube is expanded so that the zone of yielding penetrates part way or in some instances entirely through the tube, and the internal pressure is then removed, desirable residual stresses may be produced. For certain residual stress distributions, it can be shown that upon the reapplication of pressure, the tube will not yield again until a pressure equal to the maximum expanding pressure previously applied has been reached. The elastic strength of a partially yielded tube thus may be considerably greater than that of an untreated tube of the same dimensions. This process has been used frequently in the fabrication of gun barrels and pressure vessels. It is often called "autofrettage" from the French for "self-hooping," since the partial yielding process produces desirable residual stresses at the bore similar to those produced by shrinking on the outer tubes of built-up guns.

Deformation of the tube in excess of the amount necessary to make it completely plastic is generally accompanied by large plastic strains. The assumption that the deformations of an element of material are negligible in comparison with the dimensions of the element, ordinarily made for elastic and small plastic strains, is no longer valid. For such large deformations a special analysis is required, making use of more general strain definitions (2).

For elastic strains, or for strains present in the plastic region of a partially plastic tube, the deformations are small enough so that the customary assumption that they are negligible with respect to the dimensions of a deforming element is well justified. The present discussion is restricted to a consideration of partially plastic tubes, including the elastic tube and the just completely plastic tube as limiting cases, and the ordinary expressions for strains are retained.

Several solutions have been published for the partially plastic tube subjected to internal pressure (3–5). Table I classifies them according to various assumptions used in their derivation.

As can be seen from Table I, the theory developed in this paper combines elastic compressibility of the plastic material with the distortion energy theory of plastic flow. It is applicable to a material having an arbitrary stress-strain curve and holds for any combination of internal

pressure, external pressure and end load. A double need for such a theory exists at the present time; first to permit closer design practice, and second to make possible a critical evaluation and comparison of the various more approximate solutions in use at present

Table I.

Comparison of Several Theories for Partial Yielding of Thick-Walled Tubes.

Theory	Assumed Law of Yielding	Assumed Sum of Total Strains in Plastic Region	Assumed Axial Stress or Strain	Assumed Stress Strain Law	Assumed Loading
Nádai (3)	Distortion Energy	0	$\epsilon_z = 0$	$\sigma = \sigma_0$ in plastic region	Internal Pressure, End Load such that $\epsilon_z = 0$
Duguet (4)	Shear with Friction	Indeterminate	Indeterminate	Arbitrary	Internal Pressure
Macrae (5)	Maximum Shear	Indeterminate	Indeterminate	Arbitrary	Internal Pressure
Theory given in this paper	Distortion Energy	$\frac{1-2\nu}{E}(\sigma_r + \sigma_t + \sigma_z)$	(derived)	Arbitrary	Any Combination of Internal Pressure, External Pressure, End Load

STRESS ANALYSIS FOR PARTIAL YIELDING OF A THICK WALLED TUBE.

The following assumptions are made concerning the plastic or partially plastic thick-walled tube in which the elastic strains in the plastic region are considered:

(i) The sum of the total strains is

$$\epsilon_r + \epsilon_t + \epsilon_z = \frac{1-2\nu}{E}(\sigma_r + \sigma_t + \sigma_z).$$

(ii) The ratios of the principal strain differences to the corresponding principal stress differences are equal.

(iii) The axial strain is constant throughout the tube.

(iv) There is a unique relationship between the effective stress and the effective strain.

(v) The tube is acted upon by internal pressure, external pressure and an end load.

Assumption (i) takes into consideration the compressibility of the material, since in all real materials the volume does not remain constant, but increases with increasing mean stress. In a purely elastic metal the sum of the strains is equal to $(1 - 2\nu)/E$ times the sum of the stresses. When a metal becomes plastic it is assumed that the total principal strains are made up of two components, satisfying the rela-

tionship that the sum of the plastic components of the principal strains is zero, and the sum of the elastic components of the principal strains is $(1 - 2\nu)/E$ times the sum of the principal stresses as in a completely elastic material. This leads to the expression in assumption (i) for the sum of the total strains.

Assumption (ii) can be derived from the fundamental laws of plastic flow (3). Assumption (iii) assures that plane sections of the cross section remain plane during the expansion of the tube. This must be true except at sections very close to the ends of the tube.

The effective stress at a point in a body is defined as

$$\sigma = \frac{\sqrt{2}}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}, \tag{2}$$

where σ_1, σ_2 and σ_3 are principal stresses. Similarly the effective strain is defined as

$$\epsilon = \frac{\sqrt{2}}{3}\sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2}, \tag{3}$$

where ϵ_1, ϵ_2 and ϵ_3 are the principal strains. Experience indicates that for a given material and for values of ϵ less than about 0.2 a curve of σ versus ϵ is independent of the nature of the deformation. For such values of ϵ the relationship between σ and ϵ can therefore be determined by a tension test. Assumption (iv) can be expressed as

$$\sigma = f(\epsilon). \tag{4}$$

For a long uniform thick-walled tube, the principal strains may be expressed in terms of the principal stresses as

$$\left.\begin{aligned} E\epsilon_r &= \sigma_r - \nu(\sigma_t + \sigma_z) + D[\sigma_r - \tfrac{1}{2}(\sigma_t + \sigma_z)],\\ E\epsilon_t &= \sigma_t - \nu(\sigma_z + \sigma_r) + D[\sigma_t - \tfrac{1}{2}(\sigma_z + \sigma_r)],\\ E\epsilon_z &= \sigma_z - \nu(\sigma_r + \sigma_t) + D[\sigma_z - \tfrac{1}{2}(\sigma_r + \sigma_t)], \end{aligned}\right\} \tag{5}$$

where D is a function of the radius, and is zero when there is no plastic flow. Equations (5) may be written in simpler form as

$$\left.\begin{aligned} E\epsilon_r &= \omega_1\sigma_r - \omega_2(\sigma_t + \sigma_z),\\ E\epsilon_t &= \omega_1\sigma_t - \omega_2(\sigma_z + \sigma_r),\\ E\epsilon_z &= \omega_1\sigma_z - \omega_2(\sigma_r + \sigma_t), \end{aligned}\right\} \tag{6}$$

where

$$\left.\begin{aligned} \omega_1 &= 1 + D,\\ \omega_2 &= \nu + \frac{D}{2}. \end{aligned}\right\} \tag{7}$$

Equations (5) and (6) satisfy assumptions (i) and (ii) since from these equations it follows that

$$\epsilon_r + \epsilon_t + \epsilon_z = \frac{1 - 2\nu}{E}(\sigma_r + \sigma_t + \sigma_z)$$

and

$$\frac{\epsilon_r - \epsilon_t}{\sigma_r - \sigma_t} = \frac{\epsilon_t - \epsilon_z}{\sigma_t - \sigma_z} = \frac{\epsilon_z - \epsilon_r}{\sigma_z - \sigma_r}.$$

A consideration of the equilibrium of an element of the tube shows that the following equation must be satisfied by the stresses (1):

$$r\frac{d\sigma_r}{dr} = \sigma_t - \sigma_r. \tag{8}$$

Introducing the variable x defined as

$$x = \log_e \frac{r}{R_1}, \tag{9}$$

equation (8) becomes

$$r\frac{d\sigma_r}{dr} = \frac{d\sigma_r}{dx} = \sigma_r' = \sigma_t - \sigma_r, \tag{10}$$

where the prime (′) indicates differentiation with respect to x.

Compatibility of the strains (1) requires that

$$r\frac{d\epsilon_t}{dr} = \epsilon_r - \epsilon_t \tag{11}$$

or, in terms of the variable x,

$$\epsilon_t' = \epsilon_r - \epsilon_t. \tag{12}$$

Substituting the expressions for the strains from equation (6) into equation (12) gives

$$a_t D' + \omega_1\sigma_r' + \omega_1\sigma_t' - \omega_2\sigma_z' = 0, \tag{13}$$

where a_t is one of the three quantities defined as

$$\left.\begin{aligned} a_r &= \sigma_r - \tfrac{1}{2}(\sigma_t + \sigma_z), \\ a_t &= \sigma_t - \tfrac{1}{2}(\sigma_z + \sigma_r), \\ a_z &= \sigma_z - \tfrac{1}{2}(\sigma_r + \sigma_t). \end{aligned}\right\} \tag{14}$$

As stated in assumption (iii) the axial strain ϵ_z is constant over the cross section, or

$$\epsilon_z' = 0. \tag{15}$$

Using the expression for ϵ_z from equation (6), equation (15) becomes

$$a_z D' - \omega_2\sigma_r' - \omega_2\sigma_t' + \omega_1\sigma_z' = 0. \tag{16}$$

The unique relationship between the effective stress σ and the effective strain ϵ

$$\sigma = f(\epsilon)$$

has been taken as the final condition which must be satisfied by the stresses and strains in the thick-walled tube. From equation (3),

$$\frac{9}{2}\epsilon^2 = (\epsilon_r - \epsilon_t)^2 + (\epsilon_t - \epsilon_z)^2 + (\epsilon_z - \epsilon_r)^2.$$

Differentiating with respect to x this becomes

$$9\epsilon\epsilon' = \frac{4}{E}(\omega_1 + \omega_2)(a_r\epsilon_r' + a_t\epsilon_t' + a_z\epsilon_z'). \tag{17}$$

In a similar fashion from equation (2)

$$2\sigma^2 = (\sigma_r - \sigma_t)^2 + (\sigma_t - \sigma_z)^2 + (\sigma_z - \sigma_r)^2.$$

Differentiating with respect to x gives

$$\sigma\sigma' = a_r\sigma_r' + a_t\sigma_t' + a_z\sigma_z'. \tag{18}$$

The ratio of equation (18) to equation (17) is

$$\frac{\sigma\sigma'}{9\epsilon\epsilon'} = \frac{a_r\sigma_r' + a_t\sigma_t' + a_z\sigma_z'}{\frac{4}{E}(\omega_1 + \omega_2)(a_r\epsilon_r' + a_t\epsilon_t' + a_z\epsilon_z')}. \tag{19}$$

Equation (19) can, by means of the relationships given in equations (6), (7) and (14), be reduced to

$$\phi_1 D' + \phi_2 a_r\sigma_r' + \phi_2 a_t\sigma_t' + \phi_2 a_z\sigma_z' = 0, \tag{20}$$

where

$$\left.\begin{aligned} \phi_1 &= \frac{4m}{9}(\omega_1 + \omega_2)(a_r^2 + a_t^2 + a_z^2), \\ \phi_2 &= \frac{4m}{9}(\omega_1 + \omega_2)^2 - 1, \\ m &= \frac{1}{E^2}\frac{\sigma}{\epsilon}\frac{\sigma'}{\epsilon'} = \frac{1}{E^2}\frac{\sigma}{\epsilon}\frac{d\sigma}{d\epsilon}. \end{aligned}\right\} \tag{21}$$

Equations (10), (13), (16), and (20) now form a group of four equations in the four unknowns σ_r', σ_t', σ_z' and D'. They are rewritten together as follows

$$\left.\begin{aligned} \sigma_r' &= \sigma_t - \sigma_r, \\ a_t D' + \omega_1\sigma_r' + \omega_1\sigma_t' - \omega_2\sigma_z' &= 0, \\ a_z D' - \omega_2\sigma_r' - \omega_2\sigma_t' + \omega_1\sigma_z' &= 0, \\ \phi_1 D' + \phi_2 a_r\sigma_r' + \phi_2 a_t\sigma_t' + \phi_2 a_z\sigma_z' &= 0. \end{aligned}\right\} \tag{22}$$

These equations may be solved simultaneously for σ_r', σ_t', σ_z' and D' in terms of the values of a_i, ω_i and ϕ_i, which in turn are functions of σ_r, σ_t, σ_z and D. The solutions are

$$\left.\begin{aligned}
\sigma_r' &= \sigma_t - \sigma_r, \\
\sigma_t' &= \frac{[(\omega_1 - \omega_2)a_z^2 + \omega_1 a_r a_t]\phi_2 + (\omega_2^2 - \omega_1^2)\phi_1}{[\text{Den}]}(\sigma_t - \sigma_r), \\
\sigma_z' &= \frac{(a_r - a_t)(a_z\omega_1 + a_t\omega_2)\phi_2}{[\text{Den}]}(\sigma_t - \sigma_r), \\
D' &= \frac{(\omega_1^2 - \omega_2^2)(a_t - a_r)\phi_2}{[\text{Den}]}(\sigma_t - \sigma_r),
\end{aligned}\right\} \tag{23}$$

where

$$\left.\begin{aligned}
[\text{Den}] &= -[(a_t^2 + a_z^2)\omega_1 + 2a_t a_z\omega_2]\phi_2 + (\omega_1^2 - \omega_2^2)\phi_1, \\
\phi_1 &= \frac{4m}{9}(\omega_1 + \omega_2)(a_r^2 + a_t^2 + a_z^2), \\
\phi_2 &= \frac{4m}{9}(\omega_1 + \omega_2)^2 - 1, \\
m &= \frac{1}{E^2}\frac{\sigma}{\epsilon}\frac{d\sigma}{d\epsilon}, \\
\omega_1 &= 1 + D, \\
\omega_2 &= \nu + \frac{D}{2}, \\
a_r &= \sigma_r - \tfrac{1}{2}(\sigma_t + \sigma_z), \\
a_t &= \sigma_t - \tfrac{1}{2}(\sigma_z + \sigma_r), \\
a_z &= \sigma_z - \tfrac{1}{2}(\sigma_r + \sigma_t).
\end{aligned}\right\}$$

This system of differential equations is best solved numerically for any particular problem. The numerical solution is difficult since the boundary conditions can be satisfied only by trial and error methods. (These conditions for a given value of R_1/R_0 are the external and internal pressures, the external or internal tangential strain, and the end load.) A numerical solution can be obtained by assuming first that $\sigma_r = -p_1$ at the external surface of the tube. This satisfies the condition that the external pressure shall have a value p_1. Usually the value of p_1 is taken as zero. Reasonable values of σ_t and σ_z at the surface of the tube can then be assumed. The value of D corresponding to the required value of the external tangential strain can be determined from equation (5), and the numerical solution carried out by standard methods (2, 6). It will be necessary to try several combinations of values of σ_t and σ_z in order to determine which pair satisfies the other two boun-

dary conditions (i.e. to determine which pair of σ_t, σ_z values assumed at the surface of the tube will give simultaneously a preselected bore pressure p_0 and a preselected end load F).

It is evident that some method of simplification would be desirable in order to decrease the labor of solving the system of differential equations arising in this problem.

One of the greatest simplifications possible would be to assume an idealized stress-strain curve which could be made to fit the majority of actual stress-strain curves with a good degree of approximation. This

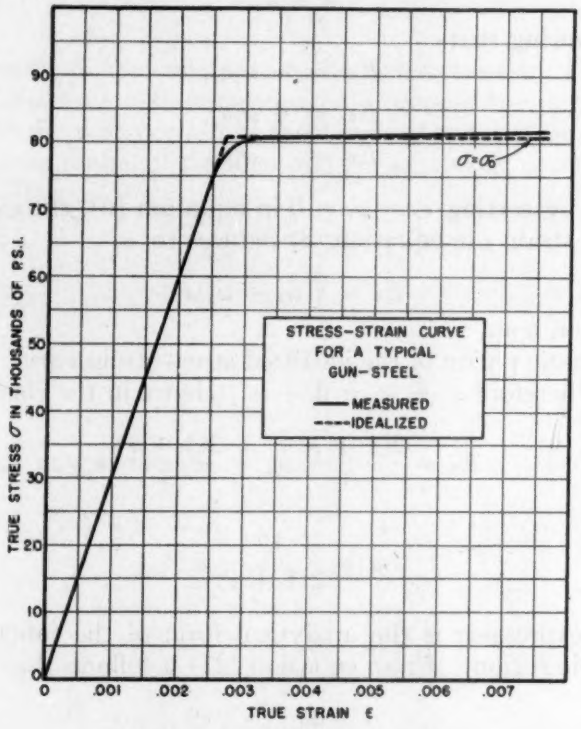


FIGURE 2

would make unnecessary a recomputation of the solution of the equations whenever a material with a different stress-strain relationship was considered. Further, a group of numerical solutions could be prepared in tabular form so that the solution for any particular set of boundary conditions could be obtained by interpolation. This method of simplification is adopted here.

Since steel is the material most generally used for thick-walled tubes, and since the stress-strain curve for steel frequently resembles the curve for a typical gun-steel given in Fig. 2, it is usually possible to simplify the stress-strain curve to two straight lines. The first, passing through the origin and having a positive slope, represents the relation-

ship in the elastic region, and the second, having zero slope, represents the relationship in the plastic region. This approximation is shown in Fig. 2. It must be borne in mind that the approximation breaks down for strains much larger than a few per cent. For partially yielded tubes, however, where the strains do not exceed this limit, the approximation is satisfactory.

The value of the effective stress σ for the tension test may be determined from equation (2) together with the fact that $\sigma_r = \sigma_t = 0$. Equation (2) reduces to

$$\sigma = \sigma_z.$$

Similarly knowing that

$$\begin{aligned} E\epsilon_r &= -\omega_2\sigma_z, \\ E\epsilon_t &= -\omega_2\sigma_z, \\ E\epsilon_z &= \omega_1\sigma_z, \end{aligned}$$

as obtained by setting $\sigma_r = \sigma_t = 0$ in equation (6), the expression for the effective strain ϵ in equation (3) reduces to

$$E\epsilon = \tfrac{2}{3}(\omega_1 + \omega_2)\sigma_z$$

for the tension test.

In the elastic region of the idealized stress-strain curve the quantity $D = 0$ and therefore $\omega_1 + \omega_2 = 1 + \nu$. Hence in the elastic region

$$E\epsilon = \frac{2(1+\nu)}{3}\sigma_z = \frac{2(1+\nu)}{3}\sigma$$

and

$$\sigma = \frac{3E}{2(1+\nu)}\epsilon.$$

The above expression is the analytical form of the function $\sigma = f(\epsilon)$ for the elastic region. From equation (21) it follows that

$$m = \frac{1}{E^2}\frac{\sigma}{\epsilon}\frac{d\sigma}{d\epsilon} = \frac{9}{4(1+\nu)^2},$$

from which

$$\phi_2 = \frac{4m}{9}(\omega_1 + \omega_2)^2 - 1 = \frac{4m}{9}(1+\nu)^2 - 1 = 0$$

and equations (23) reduce to

$$\left.\begin{aligned} \sigma_r' &= \sigma_t - \sigma_r, \\ \sigma_t' &= -(\sigma_t - \sigma_r), \\ \sigma_z' &= 0, \\ D' &= 0 \end{aligned}\right\} \qquad (24)$$

for the elastic portion of the stress-strain curve. Equations (24) integrate directly into the well-known equations for an elastic thick-walled tube subjected to arbitrary internal and external pressures and end load. Changing from the variable x to the variable r the solutions are

$$\left.\begin{aligned} \sigma_r &= A - \frac{B}{r^2}, \\ \sigma_t &= A + \frac{B}{r^2}, \\ \sigma_z &= C, \end{aligned}\right\} \tag{25}$$

where A, B and C are constants to be determined by the boundary conditions. Since $D = 0$, equations (23) become the ordinary elastic equations, for the first portion of the idealized stress-strain curve.

In the plastic region of the idealized curve, $\sigma = \sigma_z = \sigma_0 = \text{constant}$, so that

$$m = \frac{1}{E^2}\frac{\sigma}{\epsilon}\frac{d\sigma}{d\epsilon} = 0,$$

from which

$$\left.\begin{aligned} \phi_1 &= 0, \\ \phi_2 &= -1. \end{aligned}\right\}$$

Equations (23) now reduce to

$$\left.\begin{aligned} \sigma_r' &= \sigma_t - \sigma_r, \\ \sigma_t' &= \frac{(\omega_2 - \omega_1)a_z^2 - \omega_1 a_r a_t}{(\text{Den})}(\sigma_t - \sigma_r), \\ \sigma_z' &= \frac{(a_t - a_r)(a_z\omega_1 + a_t\omega_2)}{(\text{Den})}(\sigma_t - \sigma_r), \\ D' &= \frac{(\omega_2^2 - \omega_1^2)(a_t - a_r)}{(\text{Den})}(\sigma_t - \sigma_r), \end{aligned}\right\} \tag{26}$$

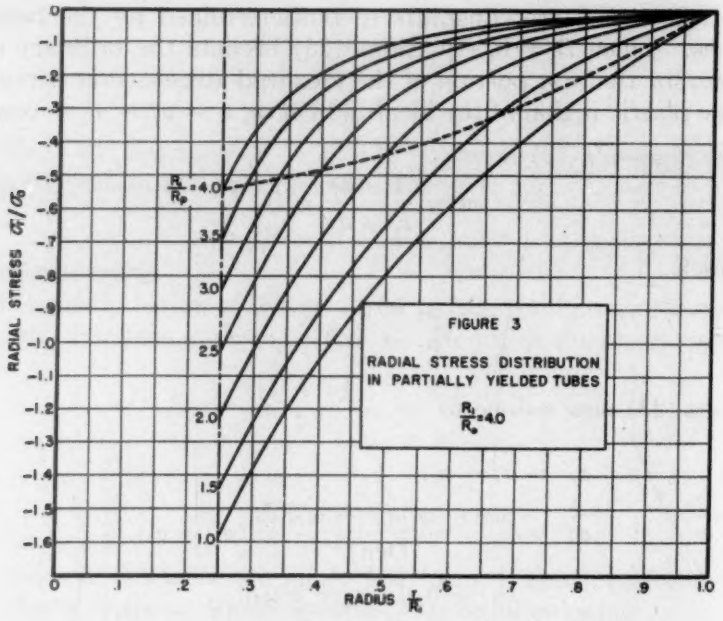
where

$$\left.\begin{aligned} (\text{Den}) &= (a_t^2 + a_z^2)\omega_1 + 2a_t a_z \omega_2, \\ \omega_1 &= 1 + D \\ \omega_2 &= \nu + \frac{D}{2}, \\ a_r &= \sigma_r - \tfrac{1}{2}(\sigma_t + \sigma_z), \\ a_t &= \sigma_t - \tfrac{1}{2}(\sigma_z + \sigma_r), \\ a_z &= \sigma_z - \tfrac{1}{2}(\sigma_r + \sigma_t). \end{aligned}\right\}$$

The system of equations (26) can be solved numerically for σ_r, σ_t, σ_z and D as functions of $x = \log_e (r/R_1)$ taking as boundary conditions

$$\text{Elastic} \left\{ \begin{matrix} \sigma_r = \sigma_r \\ \sigma_t = \sigma_t \\ \sigma_z = \sigma_z \\ D = D(=0) \end{matrix} \right\} \text{Plastic}$$

at the plastic-elastic boundary (the point of discontinuous slope in the idealized stress-strain curve).



No general solution has been found as yet for this system of equations. However, a large number of particular solutions for tubes with zero end load have been found by numerical methods, covering a range of values of R_1/R_0 from 1 to 4 for a number of depths of yielding. These solutions are arranged in tabular form in a report presented to the Bureau of Ordnance, Navy Department (7). A simple interpolation in the tables of this report gives values of σ_r/σ_0, σ_t/σ_0, σ_z/σ_0 and D as functions of x in the plastic region of partially yielded tubes for any desired values of R_1/R_0 in the range $1 \leqslant R_1/R_0 \leqslant 4$ and of R_1/R_p in the range $1 \leqslant R_1/R_p \leqslant R_1/R_0$. The stresses in the elastic region can be computed easily from the usual Lamé relationships once the stresses at the plastic-elastic boundary are known.

Although space does not permit the reproduction of these tables here, some idea as to the information they contain is given in Figs. 3–11. These figures show the stress distributions for several tubes with different ratios R_1/R_0 for a number of depths of yielding. A dashed line has been drawn in each figure in such a manner that the portion of a

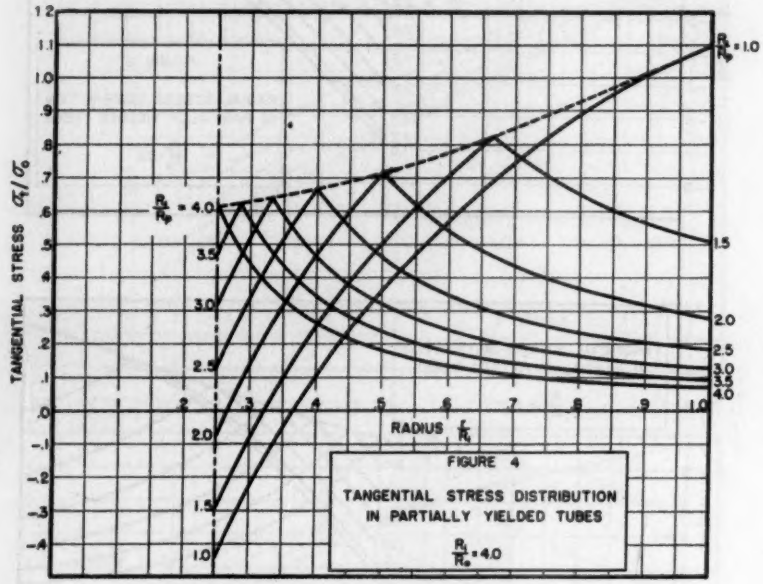


FIGURE 4

TANGENTIAL STRESS DISTRIBUTION IN PARTIALLY YIELDED TUBES

$\frac{R_1}{R_0}$ = 4.0

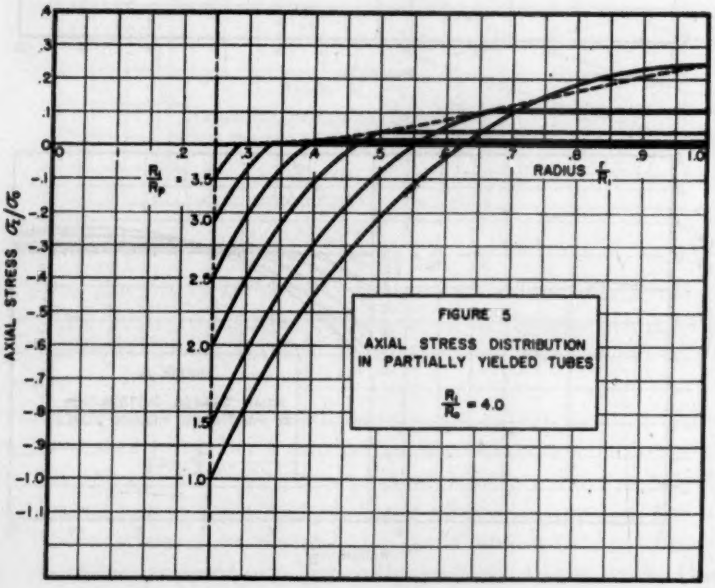
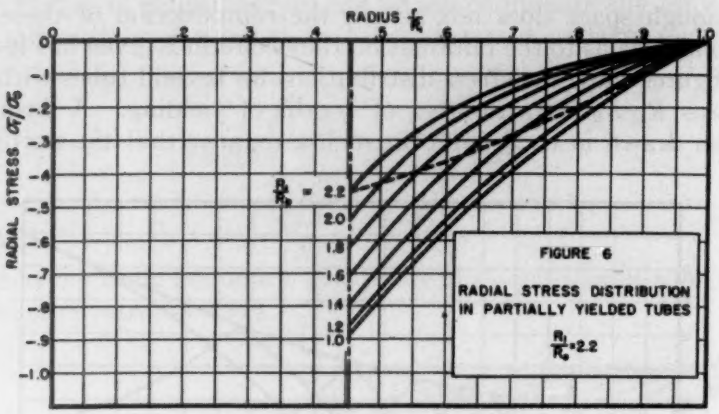


FIGURE 5

AXIAL STRESS DISTRIBUTION IN PARTIALLY YIELDED TUBES

$\frac{R_1}{R_0}$ = 4.0


RADIUS $\frac{r}{R_i}$
RADIAL STRESS σ_r/σ_0
$\frac{R_i}{R_p}$ = 2.2
2.0
1.8
1.6
1.4
1.2
1.0
FIGURE 6
RADIAL STRESS DISTRIBUTION IN PARTIALLY YIELDED TUBES
$\frac{R_i}{R_o}$=2.2

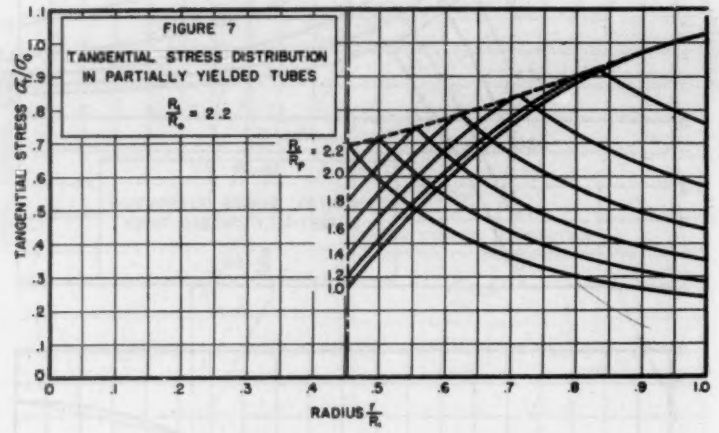
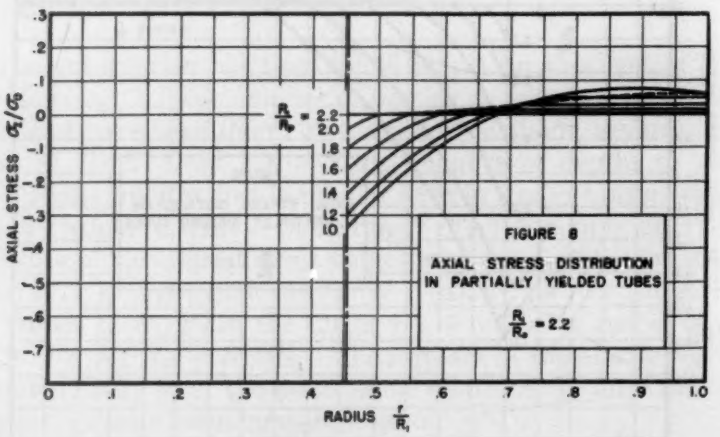
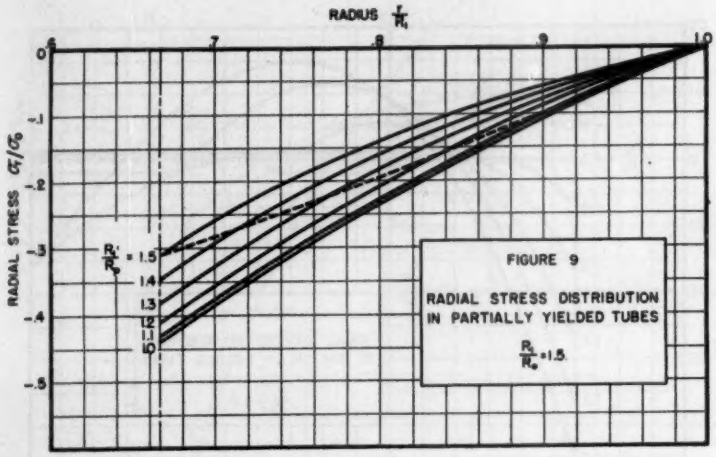
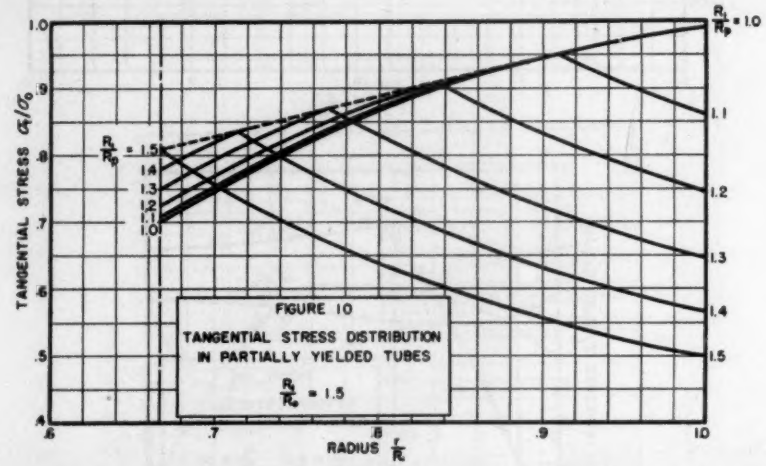


FIGURE 7
TANGENTIAL STRESS DISTRIBUTION IN PARTIALLY YIELDED TUBES
$\frac{R_i}{R_o}$ = 2.2
TANGENTIAL STRESS σ_t/σ_0
$\frac{R_i}{R_p}$ = 2.2
2.0
1.8
1.6
1.4
1.2
1.0
RADIUS $\frac{r}{R_i}$


AXIAL STRESS σ_z/σ_0
$\frac{R_i}{R_p}$ = 2.2
2.0
1.8
1.6
1.4
1.2
1.0
FIGURE 8
AXIAL STRESS DISTRIBUTION IN PARTIALLY YIELDED TUBES
$\frac{R_i}{R_o}$ = 2.2
RADIUS $\frac{r}{R_i}$

stress curve lying to the left of its point of contact with the dashed line represents the stress in the plastic region, and the portion lying to the right of the point of contact represents the stress in the elastic region.

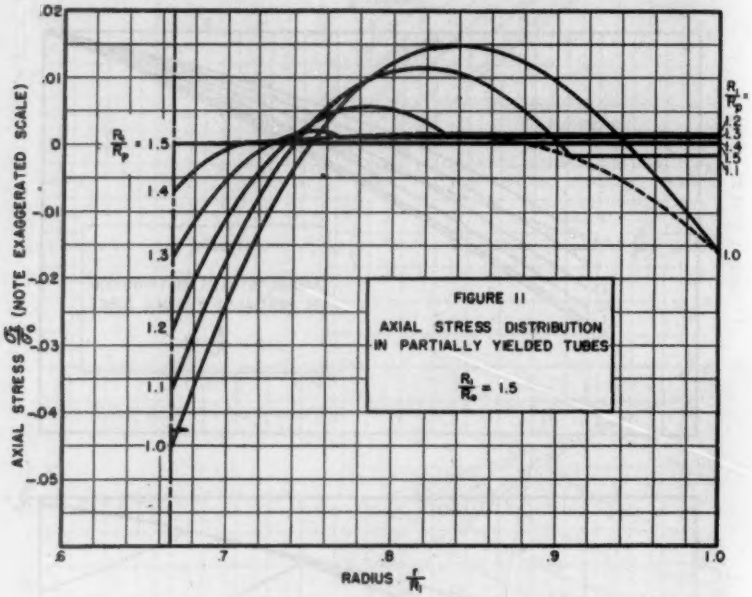


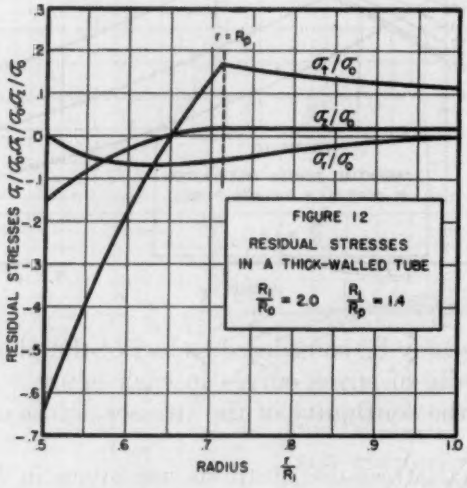


The dashed lines may be considered as loci of the plastic-elastic boundaries for the family of stress curves in each figure. It is of particular interest to note the continuity of the stresses across each plastic-elastic boundary.

A few residual stress distributions are given in Figs. 12–14. The residual stresses have been computed assuming elastic recovery of the tube when the expanding pressure is removed. This is justified in the absence of an appreciable Bauschinger effect (5), and in the absence of residual bore stresses exceeding the yield strength σ_0. For strains as small as those present in partially-yielded tubes, the Bauschinger

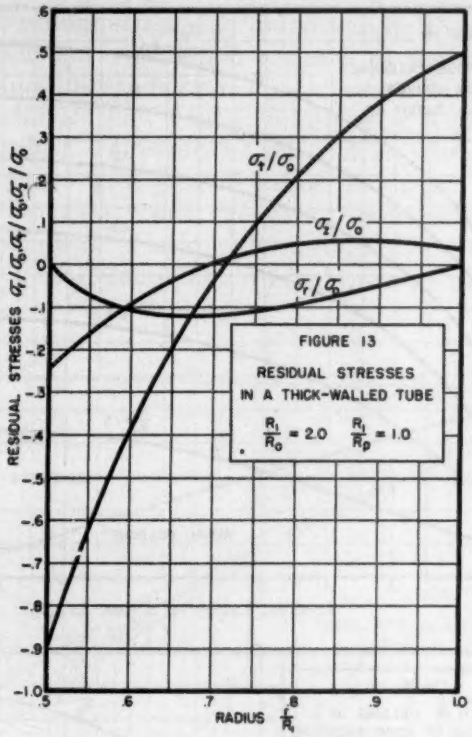
effect is frequently negligible. However, an examination of the residual stresses at the bore of the tube in Fig. 14 shows that the residual effective stress exceeds σ_0. This indicates that this tube will yield again

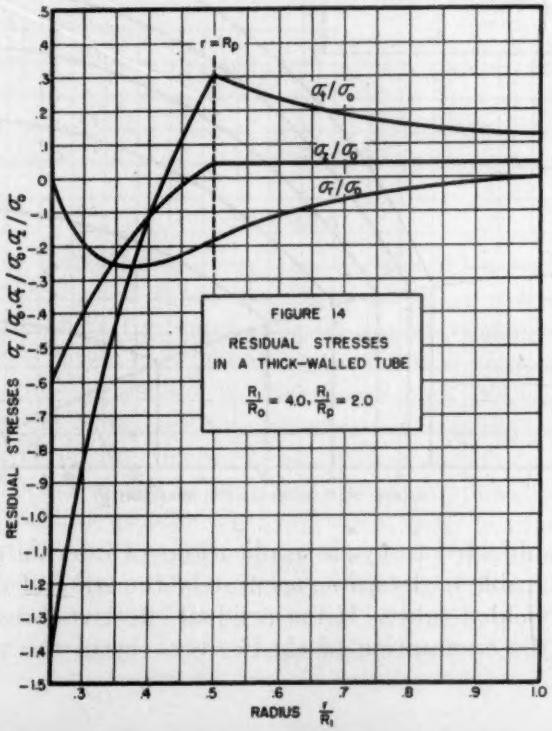


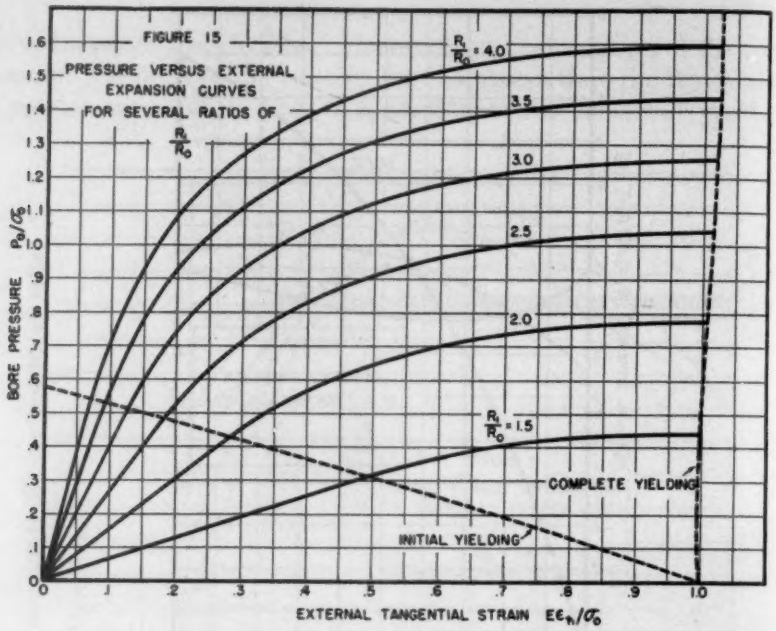


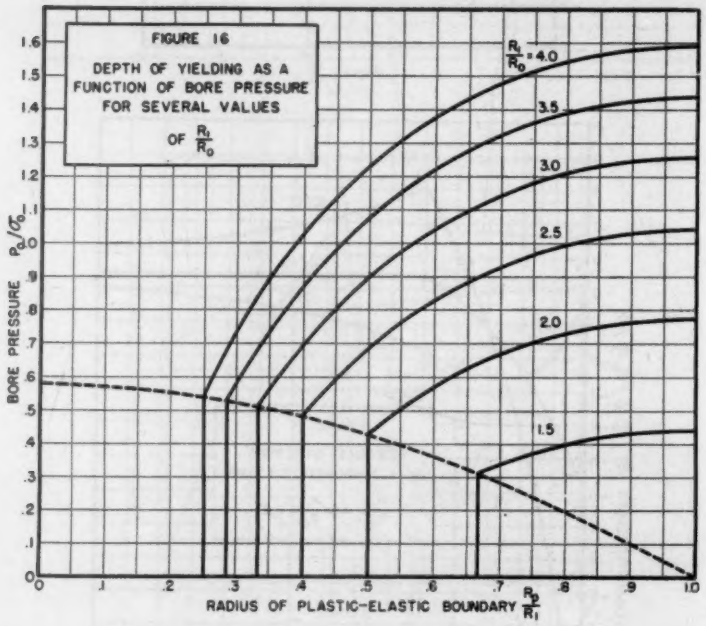
upon removal of the expanding pressure, and that the actual state of residual stress will be altered from that shown in the figure.

As it is generally inadvisable to have the metal at the bore of a tube yield again upon removal of the expanding pressure, especially when the


σ_t/σ_0
σ_z/σ_0
σ_r/σ_0
RESIDUAL STRESSES $\sigma_r/\sigma_0, \sigma_t/\sigma_0, \sigma_z/\sigma_0$
RADIUS $\frac{r}{R_i}$
FIGURE 13
RESIDUAL STRESSES
IN A THICK-WALLED TUBE
$\frac{R_i}{R_0} = 2.0 \quad \frac{R_i}{R_p} = 1.0$


$r = R_p$
σ_t/σ_0
σ_z/σ_0
σ_r/σ_0
RESIDUAL STRESSES $\sigma_r/\sigma_0, \sigma_t/\sigma_0, \sigma_z/\sigma_0$
RADIUS $\frac{r}{R_i}$
FIGURE 14
RESIDUAL STRESSES
IN A THICK-WALLED TUBE
$\frac{R_i}{R_0} = 4.0, \frac{R_i}{R_p} = 2.0$

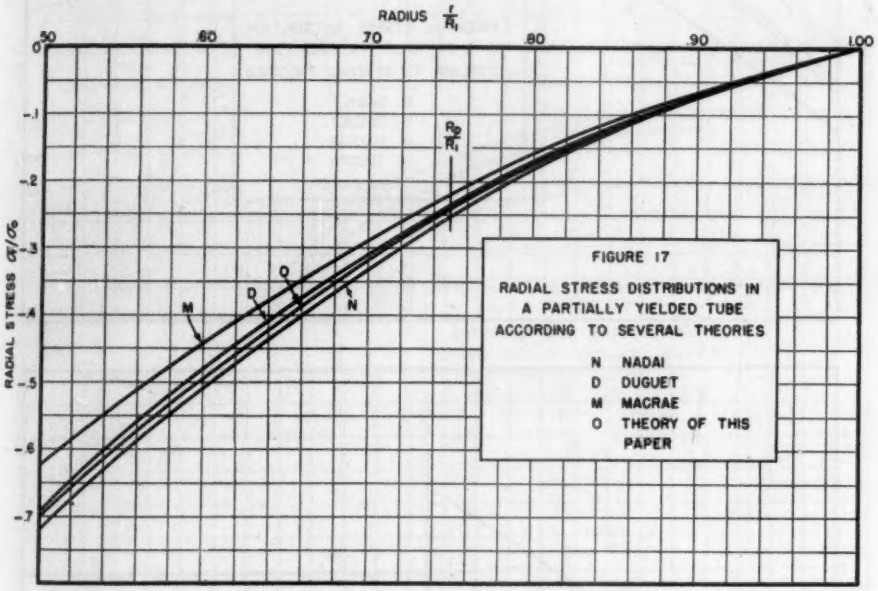




tube is to be subjected to cyclic applications of internal pressure, it is important to be able to determne accurately the state of residual stress in a partially yielded tube. If the residual effective stress at the bore computed on the assumption of elastic recovery of the tube does not

exceed σ_0, the residual stresses so found will be those actually present, and the tube will not yield upon removal of the expanding pressure. A negligible Bauschinger effect has been assumed in this discussion.

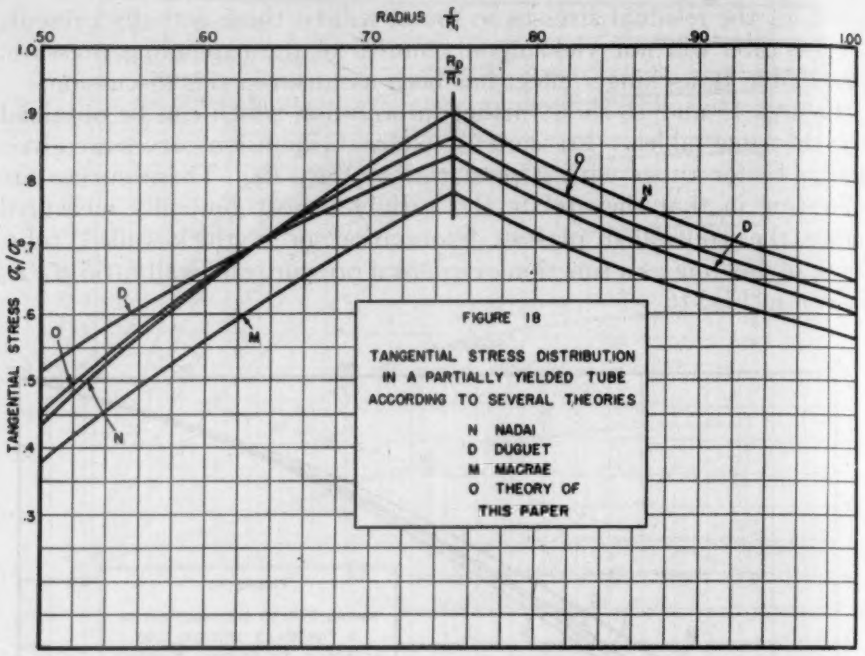
Figures 15 and 16 show further information which can be obtained from the same tables. Pressure vs. external expansion curves are given in Fig. 15 for tubes with several ratios of R_1/R_0. These curves are interesting in that they relate the variables most generally measured during the controlled plastic deformation of a thick-walled tube. Depth of yielding as a function of internal pressure and wall ratio R_1/R_0 is given in Fig. 16.

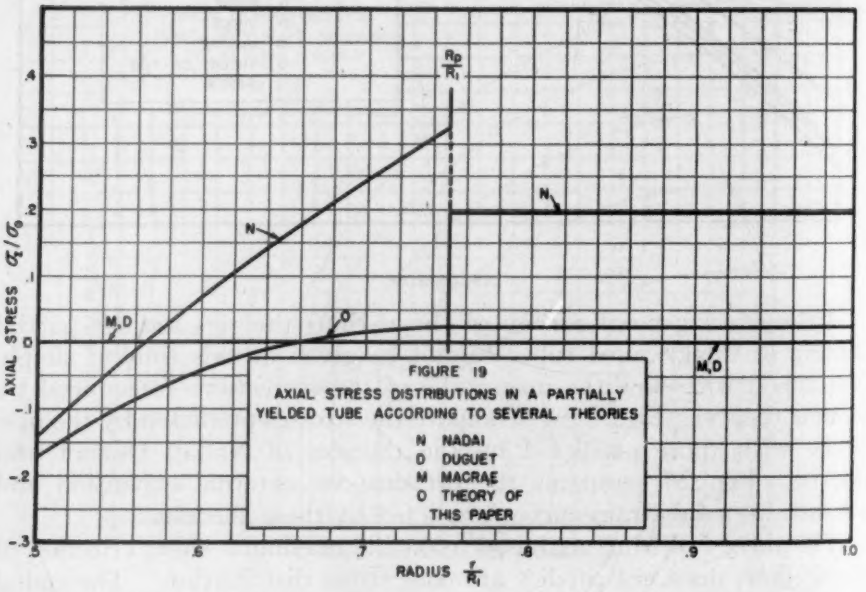


DISCUSSION.

Although a general survey of the various theories for the partial yielding of thick-walled tubes cannot be given at this time, a simple comparison will show the magnitude of the corrections introduced by the new theory. Figs. 17–19 compare the stresses predicted by the new theory with those predicted by the theories of Nádai, Duguet and Macrae. Fig. 20 compares the pressure vs. external expansion and pressure vs. axial strain curves predicted by these theories.

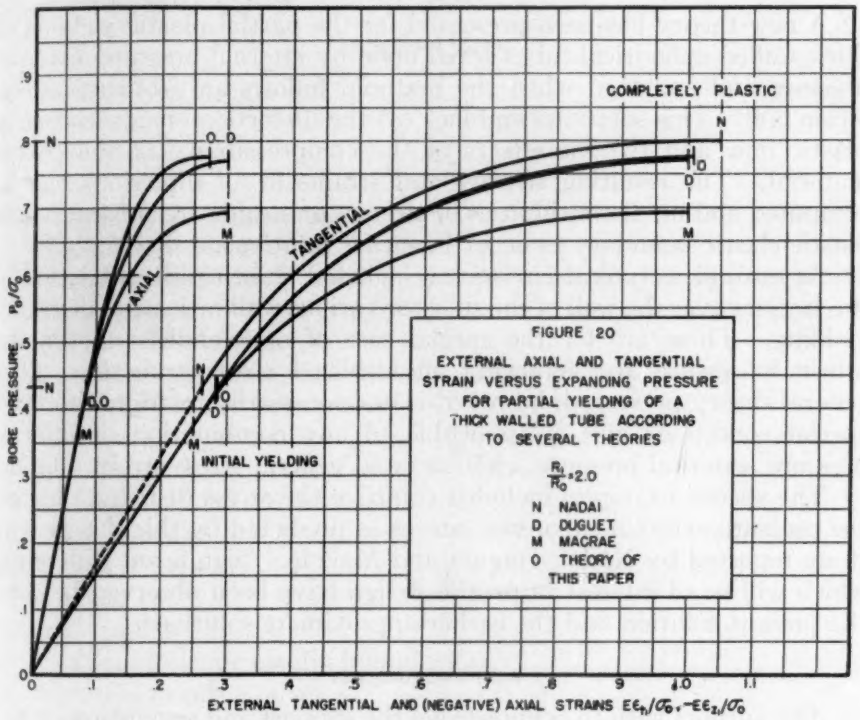
The theory of Macrae, based upon the maximum shear criterion of plastic flow, does not predict an axial stress distribution. The radial and tangential stresses corresponding to a given depth of yielding are significantly lower than those predicted by the theories based on the well-established distortion energy criterion of plastic flow, and the pressure vs. external expansion curve also falls lower for this theory.





The theory of Duguet, based on the concept of shear-with-friction similarly does not consider an axial stress distribution. The radial and tangential stresses agree fairly well with those of this paper, as does the curve of pressure vs. external expansion. A serious defect of the Duguet

theory becomes apparent when the residual stress system is considered after removal of the internal pressure. The shear-with-friction theory of yielding assumes a compressive stress for flow equal to about 1.43 times the tensile stress for flow, and indicates that residual compressive tangential stresses numerically as great as 1.43 σ_0 can exist at the bore without yielding of the tube. A design based on this figure would undoubtedly lead to further yielding at the bore upon removal of the expanding pressure.



The theory of Nádai agrees fairly well with the theory of this paper except for the axial stress distribution and the axial strain.

From this comparison, it follows that the new theory checks the order of magnitude of the radial and tangential stresses obtained by the approximate theories of Nádai and Duguet. The favorable check lends more confidence to designs based on the older theories, wherever axial stresses and strains were not involved, and where residual compressive stresses greater than σ_0 were not predicted by the Duguet theory. Where the axial stress and strain are needed, however, and where assurance is required that the tube will not yield again on removal of the expanding pressure, and in general for all precise design, the new theory provides a satisfactory solution of the problem.

It should be noted that while the comparisons made between the theory of this paper and earlier theories were for partial plastic yielding of tubes under internal pressure and zero end load assuming an idealized stress-strain law, the theory presented herein is more general than earlier ones. The present theory includes the partial yielding of tubes loaded by any combination of internal pressure, external pressure and end load, and is valid for a material following an arbitrary stress-strain law.

SUMMARY AND CONCLUSIONS.

A new theory has been presented for the partial plastic yielding of thick-walled cylindrical tubes acted upon by internal pressure, external pressure and end load when the material follows an arbitrary stress-strain law. This solution combines (a) the distortion energy theory of plastic flow and (b) the effects of the compressibility of the plastic material. The resulting stresses and strains in all directions can be computed and no discontinuities of stresses or strains are present at the plastic-elastic boundary as occur in earlier solutions.

As examples, typical curves are included showing the variation of the stresses over the wall of the tube for various wall ratios and depths of yielding. These are for the special case of open-ended tubes under internal pressure and following an idealized stress-strain law. The general theory presented, however, is not necessarily restricted to these special conditions and is applicable to any combination of internal pressure, external pressure, end load and general stress-strain relation.

The special examples included compare the stress distributions and the pressure-external expansion curves as predicted by this theory with those reported by Nádai, Duguet and Macrae. Significant differences which will be of interest in precise design have been observed between the present solution and the earlier approximate solutions.

ACKNOWLEDGMENT.

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NOMENCLATURE.

A Constant of integration.
a_r $\sigma_r - \frac{1}{2}(\sigma_t + \sigma_z)$.
a_t $\sigma_t - \frac{1}{2}(\sigma_z + \sigma_r)$.
a_z $\sigma_z - \frac{1}{2}(\sigma_r + \sigma_t)$.
B Constant of integration.
C Constant of integration.
D Variable coefficient relating stresses and strains during plastic deformation.
E Modulus of elasticity.
Constant appearing in the shear-with-friction theory of plastic flow. Generally taken as 0.7.
F End load.
m $\dfrac{1}{E^2}\dfrac{\sigma}{\epsilon}\dfrac{d\sigma}{d\epsilon}$.
p_0 Bore pressure.
p_1 External pressure.
r Radius.
R_0 Bore radius
R_p Radius of plastic-elastic boundary.
R_1 External radius
x $\log_e \dfrac{r}{R_1}$
ϵ Effective strain, $\dfrac{\sqrt{2}}{3}\sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2}$.
ϵ_1, ϵ_2, ϵ_3 } principal strains (total of elastic plus plastic).
ϵ_r Radial strain (total).*
ϵ_t Tangential strain (total).*
ϵ_z Axial strain (total).*
ν Poisson's ratio.
σ Effective stress, $\dfrac{\sqrt{2}}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$.

σ_1, σ_2, σ_3 } Principal stresses

σ_r Radial stress.*

σ_t Tangential stress.*

σ_z Axial stress.*

σ_0 Yield stress (see Fig. 2).

ϕ_1 $\dfrac{4m}{9}(\omega_1 + \omega_2)(a_r^2 + a_t^2 + a_z^2)$.

ϕ_2 $\dfrac{4m}{9}(\omega_1 + \omega_2)^2 - 1$.

ω_1 $1 + D$.

ω_2 $\nu + \dfrac{D}{2}$.

* A second subscript "0," "p," or "1" denotes the value of this variable at $r = R_0$, $r = R_p$, or $r = R_1$, respectively. Primes (′) indicate differentiation with respect to x.

INSULATION VARIABILITY.

A General Equation for Strength Distribution Curves.

BY

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An understanding of what follows requires a knowledge of the basic concepts of statistics and probability theory. Familiarity on the part of the reader with such concepts as distribution curves, standard deviation, coefficient of variability, and error function tables is taken for granted.

Earlier work by the author [1] dealt with the effect of area and inhomogeneity in lowering the breakdown voltage of systems of insulation. That paper dealt with mean values only. It was realized at that time that further progress in the field would require a knowledge of the forms of the distribution curves themselves and the changes that take place in them when units are connected in parallel. At that time, however, a solution to the problem was not clear.

The present paper deals with the actual change in strength distribution curve to be expected when units of insulation are connected in parallel. It also shows how such knowledge may be applied to specific problems.

THE GENERAL PROBLEM.

Given the breakdown distribution curve for a certain "lot" of insulating units, what kind of distribution will result when the units are tested in parallel, n units at a time?

SOLUTION.

Let $p(v)$ be the distribution function for the breakdown voltages (BDV's) of the basic insulating "units."

The probability of any unit failing above some given voltage v will then be given by the expression $\int_v^\infty p(v)dv$.

For n units in parallel the probability that all n units will fail above v is $[\int_v^\infty p(v)dv]^n$. This follows from the law of repeated independent events in probability theory.

[1] See "Insulation Variability, Its Influence in Determining Breakdown Voltages," presented at the Summer Convention of the A.I.E.E., June 1931. Also, "Breakdown Voltage as a Function of Electrode Area and Dielectric Homogeneity," JOUR. FRANK. INST., June 1931.

The probability that they will *not* all fail above v, i.e. that at least one will fail below v, then becomes $1 - [\int_v^\infty p(v)dv]^n$. But this is the condition for breakdown of the entire system of n parallel connected units. Therefore, letting $f_n(v)$ represent the distribution of breakdown voltages for the systems of n parallel connected units we have immediately the cumulative form of the required distribution

$$\int_0^v f_n(v)dv = 1 - \left[\int_v^\infty p(v)dv\right]^n. \tag{1}$$

INTERPRETATION.

This equation (1) is a basic equation. It is the starting point for the study and testing of inhomogeneous insulation. Its appearance, for a series of specific values of n, is shown in Fig. 1.

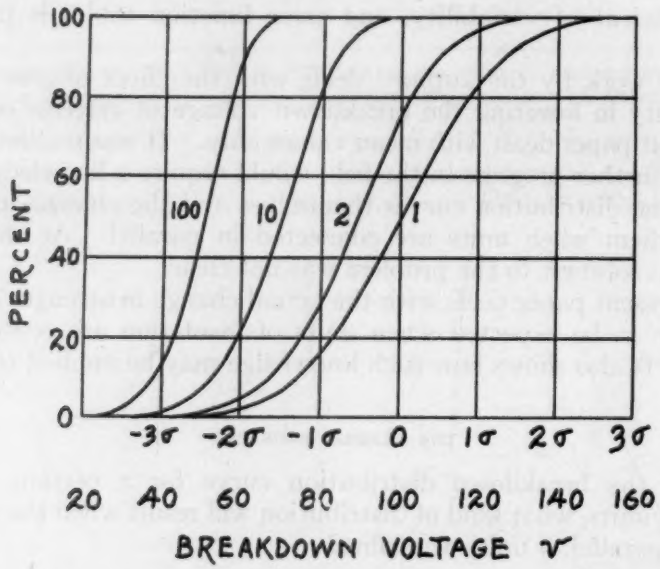

FIG. 1. Equation (1) plotted for different values of n for a normal distribution.

The curve marked (1) is the distribution of the parent population from which the "units" have been selected. It is a normal distribution with mean value at 0 and standard deviation σ equal to unity. Any voltage scale may be used. In this example, one extending from 0 to 160 volts has been chosen. This places the mean of the original distribution at 100 and thus allows one to read percentage changes directly. Such a particular choice makes the standard deviation $\sigma = 20$ voltage units and stands for a rather extremely inhomogeneous material. The ordinate scale is a simple percentage one. It has the usual meaning associated with cumulative distribution curves. For illustration, reading upward, from a voltage scale of 90, one strikes the original curve (1)

at a value of 30 per cent. This means that if the potential is raised to 90 volts, thirty per cent. of the original units will have failed. Similarly, a voltage of 100 corresponds to a percentage of 50, meaning that by the time a voltage of 100 is attained, one-half of the original units can be expected to have failed. Sixty-nine per cent. will have failed at 110 volts, 85 per cent. at 120, and practically all units will have failed at 150 volts.

Curves in Fig. 1 marked (2), (10), and (100) have been computed from eq. (1). Curve (2) represents the distribution of failures when samples are selected from the original lot and tested to destruction in parallel groups taken two at a time. Curves marked (10) and (100)

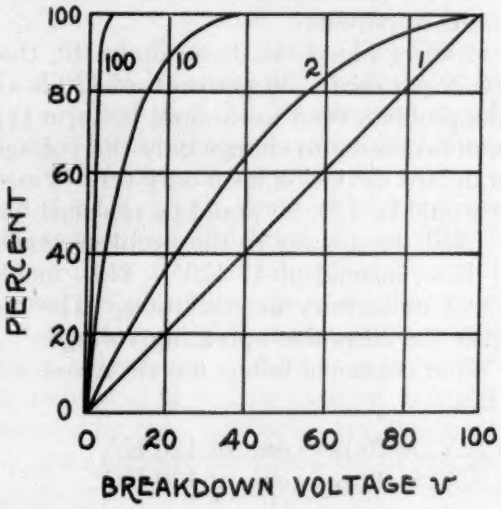

FIG. 2. Equation (1) plotted for different values of n for a uniform distribution.

are for systems of 10 in parallel and 100 in parallel respectively. Percentage of failures at different voltages may be picked off the curves as in the illustration for the original units. For example, at 60 volts approximately five per cent. of the two unit systems will have failed, twenty-one per cent. of the ten unit systems, and ninety per cent. of all the 100 unit systems. As remarked above the 60 volts might represent kilovolts or any other unit desired in a specific problem.

It is interesting to note that the derived distributions are no longer normal. This is the first time this has been shown.

Fig. 1 showed what happened in the case of an originally normal distribution. Fig. 2 shows what happens to the derived curves when the original lot is *not* normal. As a specific case a *uniform* distribution of original units extending from 0 to 100 was chosen. The curves marked (2), (10) and (100) in Fig. 2 give the results for breakdown tests made on systems of two units in parallel, ten units in parallel, and one

hundred units in parallel respectively. The curve marked (1) is the original uniform distribution. Here again it is clear that the derived curves are quite changed from the original form. As before, the voltage scale may be shifted in any way desired.

APPLICATIONS.

The general equation can be used to answer specific questions of the following type.

Problem 1. Units of insulation were tested for breakdown strength with the following results: Mean BDV 120 K.V., standard deviation 24 K.V., coefficient of variability 20 per cent. At what operating voltage will there be ten per cent chance of failure for systems consisting of 100 of these units in parallel?

Using curve (100) of Fig. 1, at the ordinate 10, the voltage is read as approximately 39 per cent. 39 per cent. of 120 is 47 K.V. *Ans.*

Note: Had the problem been for a more uniform type of insulation, it would have been necessary to change only the voltage scale of Fig. 1. Thus, had the standard deviation been only 6 K.V. instead of 24 K.V., 100 on the chart would be 120, 80 would be replaced by 114, 60 by 108, 40 by 102, etc. And the answer to the problem would have been approximately 101 K.V. instead of 47 K.V. This, incidentally, demonstrates the effect of uniformity in insulation. The more uniform the material the higher the allowable operating voltage.

Problem 2. What chance of failure has the above system if operated at 84 K.V.? 36 K.V.?

Solution: 84 K.V. is 70 per cent. of 120 K.V.
36 K.V. is 30 per cent. of 120 K.V.

Here again the answers can be read directly from curve (100) of Fig. 1 at 70 and 30 on the breakdown voltage scale. They are approximately 100 per cent. or certainty for the first case of 84 K.V. and 2 per cent. for the second case of 36 K.V. This means the odds are roughly 50 to 1 that the system will not fail at an operating voltage of 36 K.V.

It can be seen further that chance of failure becomes negligible at operating voltages lower than 25 per cent. or 30 K.V.

SUMMARY.

The equation developed here is fundamental for an understanding of the behavior of inhomogeneous insulation, especially when the insulating material is of considerable extent in the form of continuous flat sheets, or cables, or discontinuous form as in pole-line construction. The "unit" may then be measured in terms of area in the case of sheets, and in terms of length in the case of cables. It is of basic importance in insulation research, testing, and operation. It furnishes the logical basis for the establishing of safety factors.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

STANDARDIZATION OF THE pH SCALE.

The pH unit, used to express numerically the degree of acidity or alkalinity of aqueous solutions, may be defined in a number of ways, each resulting in a slightly different value for the pH of a given solution. Consequently, several pH scales, based upon various definitions, have met with equal favor among chemists. In view of the increasing need in science and industry for accurate determinations of acidity, the National Bureau of Standards is recommending the universal adoption of a single standard pH scale, analogous to the International Temperature Scale. It is proposed that the pH assigned to solutions of buffer substances distributed by the Bureau as Standard Samples be taken as the fixed points on this standard scale.

In the preparation of many commercial products—for example, paper, textiles, dyes, ceramics, and beer—the rapidity and efficiency of the processes depend upon accurate control of the acidity or alkalinity of aqueous solutions. Such control is now a regulatory requirement in the preparation of certain medicines and in the manufacture of paper and leather for the Government. In sugar manufacture, the inversion of sucrose can be regulated at will or avoided entirely by holding the acidity within certain limits. Similarly, regulation of the acidity of electroplating solutions permits the character of the deposit to be controlled. Another application of particular importance is the avoidance of corrosion and embrittlement of boiler walls and tubes by regulation of the acidity of boiler water. The widespread losses due to underground corrosion are likewise effectively curbed in many cases by proper adjustment of acidity.

The several convenient pH meters now available commercially enable precise determinations of pH values in such varied media to be made with ease and rapidity, but these values are based upon a scale fixed by the pH assigned to the standards with which the instrument has been calibrated. The differences among scales of pH are the direct result of different procedures, definitions, and assumptions employed in arriving at the pH of the standard. The pH may be defined in one instance as the negative logarithm of the hydrogen-ion concentration or, again, of the effective concentration or "activity" of this ion. Often the pH value as defined by Sorensen in terms of the electromotive force of a galvanic cell with hydrogen and calomel electrodes is chosen. Al-

*Communicated by the Director.

though the differences among these scales rarely exceed 0.1 unit, the need for greater accuracy makes desirable the general adoption of a single series of consistent pH standards.

In an effort to encourage standard procedure in pH measurements, the National Bureau of Standards is now supplying four buffer materials in the form of Standard Samples of certified purity. These substances are acid potassium phthalate, potassium dihydrogen phosphate and disodium hydrogen phosphate (intended to be used together), and borax. They are being distributed at the rate of several hundred samples annually. The certificates furnished with these compounds specify the pH of certain aqueous solutions of the sample, which can provide fixed points on a pH scale.

In order to assign exact values to these fixed points, it was necessary to set up a scale based upon some suitable definition of pH. A consideration of the advantages and limitations of several scales led to a choice of a modified activity scale as most convenient and practical for general use. Although the activity of a single ionic species can be simply defined only in very dilute solutions, the influence of the hydrogen-ion activity in chemical equilibria is of far-reaching importance.

The pH of the NBS standards is derived from measurement of the electromotive force of cells without liquid junction, in which they are used as electrolytes. These cells are specially designed, utilizing the highly reproducible hydrogen and silver-silver chloride electrodes. Computation of pH is based upon several reasonable assumed relationships between ionic activities and mean activities. These assumptions are found to give identical values for dilute solutions. The scale thus obtained approaches a true scale of activity for solutions of low concentration; at higher ionic strengths it is best regarded as a consistent scale which necessarily rests upon an assumption not subject to experimental proof.

AUTOMATIC IONOSPHERE RECORDER.

A new instrument for automatic recording of ionospheric phenomena, which are of great practical importance in radio propagation, is now in operation at the Bureau's ionospheric research station at Stirling, Virginia. The prototype model of the recorder, developed by the Bureau's Central Radio Propagation Laboratory, was completed in time to participate in the recent Army Air Forces National Geographic Society Eclipse Expedition in Brazil. In addition to special records during the May 20 eclipse of the sun, valuable data were obtained on general ionosphere conditions. Plans are being made to install these recorders in all the ionosphere stations operated by the Bureau.

Long-distance radio transmission would be impossible if it were not for the ionosphere, a series of ionized layers in the atmosphere 50 to 250

miles above the earth that reflect radio waves back to earth. The new recorder will provide automatic and continuous measurements of the heights of the various layers and of critical frequency, that is, the maximum frequency that is reflected back to earth rather than passing off into space.

The Model C2 recorder was designed to utilize the so-called multi-frequency technic of investigating the layers of the ionosphere. It is the first completely automatic recorder to use in continuous heavy duty the heterodyne pulse transmitter arrangement recently described by P. G. Sulzer.[1] In this arrangement the entire frequency range from 1.0 to 25.0 Mc./s. is covered continuously without switching bands. The receiver is tuned with the transmitter throughout its frequency range. Pulse transmissions are used similar to those employed in radar except with varying probing frequency. The frequency is plotted against the time delay of the echoes from the ionosphere, an interval that corresponds to twice the height of reflection. One sweep in frequency from lower to upper limits produces each ionosphere record in a time interval of as little as 7½ seconds.

Two oscilloscopes provide indications corresponding in essential detail to the radar "A" and radar "B" scans. A special 35-mm. camera, driven by the motor that operates the transmitter oscillator, makes a continuous film record of the indications of the main oscilloscope. As the image of the sweep line is oriented at right angles to the direction of film travel, the graph of height reflection (time delay) versus frequency is recorded on height scales of 500, 1,000, and 4,000 kilometers. Simultaneous visual observation is possible by means of the monitoring oscilloscope.

A standard 16-mm. motion picture camera is provided to photograph the monitoring oscilloscope. When a series of 16-mm. film records, made at 7½ or 15 second intervals over a long period of time, are projected, a striking accelerated version of the changes taking place in the ionosphere is observed.

The main cabinet of the new ionosphere recorder houses no moving parts, but encloses the transmitter mixer, transmitter wide-band amplifier, receiver mixer, receiver intermediate frequency unit with detector and video stages, pulsing and keying circuits for the transmitter, the transmitter fixed frequency unit, and all the power supplies for these units.

A smaller cabinet on rollers contains the recording and monitoring oscilloscopes, the recording 35-mm. camera, the variable frequency oscillator, the camera and oscillator drive motor, switches and their associated control, time base and sweep-generating circuits, and the necessary power supplies.

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[1] See "Ionosphere Measuring Equipment" by P. G. Sulzer, July *Electronics*, 1946.

Other features of the Model C2 recorder are: a pulse receiver having large dynamic range and a differentiating circuit to minimize interference by CW (continuous wave) and broadcast stations, provision for automatic operation by a clock completely independent of power-line frequency, the use of hermetically sealed units to insure reliable operation in many climates, and regulation of every important d.-c. voltage to provide stability of power source.

The replacement of the old recording units in the ionosphere station network with an automatic and continuous recorder will assure a more comprehensive and reliable flow of worldwide ionosphere data into the National Bureau of Standards. To date, the principal emphasis in the study of ionospheric phenomena has been the practical use of the information in the prediction of radio propagation conditions. However, data on the ionosphere, which may be considered as a strategically located astrophysical radiation laboratory, will reveal more and more of the characteristics of radiation from the sun, and the physical conditions of the outer atmosphere, its temperatures and densities, mean free paths of electrons, recombination processes, and geomagnetic effects. These data may also lead to information on air circulation and other phenomena in the lower atmosphere (stratosphere and troposphere).

LIGHT WAVE OF MERCURY 198 AS THE ULTIMATE STANDARD OF LENGTH.

A new and better standard of length now exists in the wavelength of green radiation of mercury 198, an isotope transmuted from gold by neutron bombardment. In precision, reproducibility, and convenience, the new standard is superior to both the standard meter and the red line of cadmium, according to recent investigations by Dr. William F. Meggers of the National Bureau of Standards. Preliminary measurements by Dr. Meggers have shown an accuracy of one part in a hundred million of relative values and one part in a billion is theoretically possible.

Since 1889 the world's standard of length has been the "meter" distance between two lines on a platinum-iridium bar at the International Bureau of Weights and Measures in France. Fundamental measurements throughout all of science and industry are based on this standard, but it has several disadvantages. First, line standards are unsuitable in certain fields of measurement. Second, the intrinsic nature of lines ruled on surfaces—such lines are in effect small furrows—limits the precision attainable. Third, the meter is not readily reproducible.

Primarily because the standard meter does not afford sufficient precision in some fields, the red line of cadmium has been universally used for many years for precise measurements. However, the cadmium standard also has serious disadvantages. First, there is a fine structure

in the red radiation which prevents the line from being as sharp as desirable and thus limits the precision possible. Second, the cadmium standard requires excitation in a furnace which entails unwanted broadening of the spectral line because of relatively high temperature.

The green line of mercury 198 has none of the disadvantages of either the meter or the red line of cadmium. The normal human eye is far more sensitive to green than to red, an important consideration in visual adjustment of the interferometer with which lengths are measured and compared. All other characteristics desirable in a light wave standard—such as ability to be reproduced, absolute sharpness of the wavelength, intensity of the spectral line, life and convenience of maintenance—are possessed to a greater extent by mercury 198.

The future refinement of physical optics—for example, an accurate determination of the velocity of light—and the improvement of mechanical processes—for example, the ruling of better diffraction gratings—are dependent on the production and adoption of an ultimate standard of length superior both to the meter bar and to the wavelength of red radiation from cadmium. The nuclear reaction that now makes possible large scale transmutation and manufacture of pure elements not found in nature will also produce any desired quantity of the pure mercury from gold, and thus provide a material for a spectroscopic light source that emits light waves much more monochromatic than any emitted by natural elements. Theoretically, mercury isotope 198 should show interference patterns with retardations exceeding a million waves, and because it is possible with monochromatic lines to measure one-one thousandth of a wave, it is probable that the relative value of Hg^{198} wavelengths may eventually be determined with an accuracy of one part in a billion.

As long ago as 1927, the National Bureau of Standards recommended that the International Conference of Weights and Measures adopt a light wavelength, that of red radiation from cadmium vapor, as the primary standard of wavelength, and that the meter be defined in terms of this wavelength. The Conference objected that such a definition of the meter would menace the metric system, and explained that it was not a question of giving a true relation between the meter and the wavelength, but only a metric value of the latter which could be modified by future experiments. Strictly speaking, the world's primary standard of length is still the distance between two relatively wide lines drawn on a metal bar, despite the fact that practically all precise measurements of lengths in the 20th century have been made, and will continue to be made, with light waves.

The most monochromatic spectral lines are emitted by massive slow-moving atoms, and because mercury atoms are nearly twice as heavy as cadmium atoms and can be excited to radiate at less than half the absolute temperature, mercury lines are less than half as wide as

cadmium lines, other things being equal. Wavelengths from natural mercury cannot be used as standards of length because natural mercury consists of a fixed mixture of seven isotopes with atomic masses of 196, 198, 199, 200, 201, 202, and 204, and each isotope emits one or more spectral components none of which are exactly coincident. Consequently, the green line of natural mercury has sixteen components.

Because the effective wavelength of such a complex line observed interferentially varies with the phase relations of the various components, it is imperative to avoid complex lines in selecting a natural standard of length. This objectionable feature of mercury lines could be removed if a single isotope, for example Hg^{204}, could be separated from the rest, but up to the present it has not been practicable to isolate an isotope of natural mercury in sufficient quantity to make satisfactory lamps. However, this goal has now been achieved by transmuting gold (Au^{197}) into mercury (Hg^{198}). The feasibility of doing this was first demonstrated in 1940 by J. Wiens and L. W. Alvarez who reported that bombardment of gold by neutrons from a 60-inch cyclotron at the University of California produced enough mercury to be detected spectroscopically.

In 1942, the National Bureau of Standards purchased forty ounces of proof gold and enlisted the cooperation of the University of California to expose this gold to neutrons for one or more years. Unfortunately, World War II interrupted the experiment and only submicroscopic quantities of artificial mercury were made. The prospects were very discouraging until, near the end of the war, there were rumors of a secret source of neutrons thousands of times more effective than the largest cyclotron. In 1945, the National Bureau of Standards gold was transferred from California to Tennessee. The treatment this gold received was not disclosed but a year later the Bureau distilled from it about 60 milligrams of mercury, which was found from spectroscopic tests by Dr. Meggers, chief of the Bureau's Spectroscopy Section, to be pure Hg^{198}. Anticipating a considerable demand for Hg^{198} lamps, the Bureau has requested the Atomic Energy Commission to bombard some more gold with neutrons to produce one or more grams of Hg^{198} within a year. In the meantime, the available Hg^{198} has been used by Dr. Meggers in the preparation of several types of lamps which are being studied to determine the one most suitable for adoption as a standard.

In the design of a Hg^{198} lamp that will emit radiations suitable as ultimate standards of length a maximum is desirable in each of the following five characteristics: (1)monochromaticity, (2) reproducibility, (3) intensity, (4) life, and (5) convenience. It is perhaps obvious that some of these requirements conflict with others, and that it will be necessary to make compromises. It appears probable that either electrodeless tubes or Geissler tubes (similar to the ubiquitous luminous

signs), containing several milligrams of Hg^{198} and a small amount of argon gas will be useful for accurate measurements.

Employing an electrodeless lamp excited by high frequency radio waves, preliminary values of the wavelengths of a dozen Hg^{198} lines, ranging from the ultraviolet (3341 A) to the yellow (5791 A), have been measured by Dr. Meggers. Though publication of the observed wavelengths of Hg^{198} will be deferred until final values are in hand, these preliminary values, when tested by the combination principle of spectroscopy, appear to be correct within one part in one hundred million, whereas the best measurements made with natural mercury exhibit deviations of one part in one hundred thousand, due, no doubt, to the falsification of the wavelengths by the complexity of the lines.

Although cadmium and mercury are divalent chemical analogues, and therefore exhibit relatively simple and similar atomic spectra, whatever differences exist are invariably in favor of mercury. For example, the brightest line in the cadmium spectrum occurs in the blue-green (5086 A), whereas the mercuric analogue is in the green (5461 A) nearly coincident with the maximum sensitivity of the normal human eye. The red wave of cadmium (6438 A) is intrinsically only one-tenth as intense as the strongest line (5086 A), and is further handicapped by the fact that the eye is only one-seventh as sensitive for red as for green. Thus for the visual adjustment of interferometers the green line of mercury is seventy times as intense as the red line of cadmium. The mercury analogue of the cadmium red line is a yellow line (5791 A) which is always accompanied by another yellow line of shorter wavelength (5770 A) but nearly equal intensity. This yellow pair of mercury lines produces interference coincidences at intervals of 275 waves, and is happily heuristic for the whole order of interference without counting any fringes; it has no convenient counterpart in cadmium.

Mercury is the only heavy stable element that has an appreciable vapor pressure below zero degrees Centigrade, and therefore is unique among all elements in radiating, at low pressure and temperature, a relatively simple spectrum of extremely sharp lines provided isotopic structure is eliminated. The green line of mercury, rejected by Michelson 55 years ago on account of complex structure, has finally, by the production of mercury 198, been freed of its seven-isotope curse, and the green line of Hg^{198} now stands alone as the most nearly ideal standard wavelength that can ever be obtained from any atoms, natural or artificial. Coupled with the fact that adequate quantities of absolutely pure Hg^{198} are now obtainable by neutron bombardment of gold in chain-reacting piles, the unique properties of Hg^{198} force the conclusion that a progressive scientific world will eventually adopt the wavelength of green radiation (5461 A) from Hg^{198} as the ultimate standard of length.

The meter unit, the present unit of length, was created about 1790 to represent one-ten millionth of the earth's quadrant. In 1827, some natural philosophers meeting in Paris agreed that the meter could not be reproduced if the form of the earth were changed by collision with a comet. A Frenchman, Jacques Babinet, then proposed a light wave in a vacuum as a natural unit of length independent of the earth's dimensions. Later the same thought was expressed by German, Dutch, and British scientists, but the first pratical results must be credited to Americans, A. A. Michelson and E. W. Morley, who, in 1887, outlined "A method of making the wavelength of sodium light the actual and practical standard of length." Their method, involving the use of the optical interferometer devised by them for their celebrated experiments on the relative motion of earth and ether, consisted of the measurement of a length and the counting of an equivalent number of interference fringes.

In 1889, Michelson and Morley described in detail a method of measuring the meter in light waves, and predicted that the brilliant mercury green line would in all probability be the wave to be used as the ultimate standard of length. Searching systematically for the radiation best suited as an ultimate standard, Michelson discovered in 1892, that the green light of mercury is complex, and discarded it in favor of the red light of cadmium. These classic investigations promptly led to Michelson's invitation to the International Bureau of Weights and Measures, where he performed his celebrated determination of the relation between the meter and the wavelength of cadmium red radiation. In the succeeding forty years Michelson's experiment was repeated a half dozen times and his result has been amply confirmed, considering the fact that the lines on the meter bar are ten to twelve wavelengths wide.

Indeed, it is the character of ruled lines themselves which limits the accuracy of wavelength-meter intercomparisons and there is therefore hardly any point to measuring the wavelengths of Hg^{198} lines relative to the meter. The wavelength of Hg^{198} green light can readily be measured relative to cadmium red light from ten to one hundred times more accurately than either relative to the meter. Adoption of the present provisional relation as exact, and subsequent substitution of Hg^{198} green for cadmium red appears to be the logical and expeditious approach to a better standard of length.

THE FRANKLIN INSTITUTE

ANNUAL MEETING, WEDNESDAY, JANUARY 21, 1948.

The Annual Meeting of The Franklin Institute was held on Wednesday evening, January 21, 1948, Mr. Richard T. Nalle, President, presiding. There were approximately 200 persons present.

The President stated that the minutes of the Stated Monthly Meeting for November were printed in full in the December issue of the JOURNAL OF THE FRANKLIN INSTITUTE. On motion, duly seconded, they were approved as printed.

The President also stated that the annual report from the Board of Managers, due to be presented at the Annual Meeting, would be presented at an adjourned meeting in the spring.

The report of the Secretary for the month of December was as follows:

Membership

Active	33
Associate	21
Student	36
Total Membership as of December 31	5424

The President then called on Dr. George S. Crampton to give the report of the tellers on the annual election for members of the Board of Managers, who were to serve for a period of three years.

Hiram S. Lukens
Orus J. Matthews
Morton Gibbons-Neff
George Wharton Pepper
M. M. Price
Charles S. Redding
James S. Rogers
Clarence Tolan, Jr.

Dr. George S. Crampton, Professor Charles M. Gay, and Mr. Ralph McClarren were the tellers for the election returns. The President declared the above members as duly elected to the Board of Managers. The President expressed his appreciation to the tellers for their service to the Institute.

The President then presented the speaker of the evening, Dr. W. F. G. Swann, Director of the Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa. Dr. Swann spoke on "The Place of Science in Our Civilization." He traced the influence of science in its effect on the normal activities of man, its effect in removing the causes of unhappiness, and in providing him with more leisure. This raised the question as to how man can occupy himself with that leisure for the greatest good of the world and for his own happiness. Dr. Swann dealt also with the relative roles of utilitarianism and non-utilitarianism in the fields of man's activities, with the end point—the greatest happiness for all—as the ultimate objective.

HENRY B. ALLEN,
Secretary.

BY-LAWS.

ARTICLE I.

STOCK.

Section 1. The Real and Personal Estates of the Institute as held upon the First day of January, One Thousand Eight Hundred and Eighty-one, shall be valued at One Hundred Thousand Dollars, and shall be represented by Ten Thousand Shares of Stock of the par value of Ten Dollars each. Said shares shall be divided into two classes, *viz.:*

First Class. Shares not registered for use: on which no annual payment shall be charged or collected, and the holders thereof shall not have the privileges of members of the Institute, but may, if of legal age, vote at any annual election for managers upon the payment of One Dollar upon each share of stock on which they may desire to vote; provided, however, such shares have been held by the same person at least three months before such election.

Shares of the First Class may be converted into shares of the Second Class at the pleasure of the owners, provided the transfer be approved by the Board of Managers; but, when once so converted, they shall always continue in the Second Class.

Second Class. Shares registered for use: on which Twelve Dollars per annum shall be due and payable from resident members in advance on the first day of October in each year, except as hereinafter provided.

Non-resident holders of Second Class stock shall pay an annual fee of Five Dollars.

Section 2. The holders of Second Class stock shall be entitled to the use of the library, lectures and reading-room, and, if of legal age, to all other privileges of membership in the Institute, so long as they make the annual payment in advance; and shall, on the payment of One Dollar therefore, be entitled to a Certificate of Membership.

Section 3. If the annual dues for successive years remain unpaid at the expiration of two and a half years on any share of stock of the Second Class, such shares shall then become forfeited to the Institute; but such forfeiture may be remitted by a unanimous vote of the Board of Managers.

Section 4. Stock of the Second Class may be held in trust for persons not of legal age, and shall be liable to the payment of only one-half the annual fees due upon stock of Second Class held by persons of legal age; provided, that when such minors arrive at legal age, new certificates, subject to the full annual contribution, shall issue on payment of the customary fee.

Section 5. Certificates for the First Class stock may be issued for any number of shares in a single certificate; but every certificate for the Second Class shall be for one share only.

Section 6. No share of stock in the Second Class shall be transferred until all arrearages and fines are paid, and all books and tickets returned, and the transfer approved by the Board of Managers.

Section 7. All certificates of stock shall be signed by the President and Secretary; shall be issued by the Controller, and shall be transferable only on the books of the Institute by the owner, or his legal representative, on the surrender of the old certificate, and of a fee of twenty-five cents for each certificate.

Section 8. All subscriptions to stock shall be approved by the Board of Managers before the certificate can be issued.

ARTICLE II.

MEMBERS.

Section 1. All persons interested in the purposes and activities of the Institute and willing to further them may become members when elected by the Board of Managers, or in a manner prescribed by the Board, except as qualified by Section 6 of this ARTICLE. Membership shall consist of the following classes:

1. Student members
2. Associate members
3. Active members
4. Sustaining members
5. Honorary members

The Board of Managers may establish other classes of members, provided that the privileges enjoyed and dues paid by such other classes are not inconsistent with the privileges and dues of the classes of members specifically provided for by these By-Laws.

Section 2. Student members shall be under twenty-five years of age and shall pay annual

dues of $2.00. They shall be entitled to unlimited free admission to the Museum and the Planetarium. Student members who are fourteen or more years of age may, if endorsed by a teacher or an Active member, have the use of the library upon the payment of additional annual dues of $1.00 but shall have no voting privileges or rights to hold office.

Section 3. Associate members shall pay annual dues of $5.00. They shall be entitled to unlimited free admission to the Museum and to the Planetarium, but shall have no voting privileges or rights to hold office. Upon the payment of $5.00 additional annual dues they shall have Family privileges as defined in Section 7 of this ARTICLE.

Section 4. Active members shall be not less than twenty years of age. They shall pay annual dues of $15.00. Active members, residing permanently at a distance of fifty miles or more from Philadelphia, shall pay annual dues of $7.50. Active members shall be entitled to use the library, to receive one copy of the Journal of The Franklin Institute, to vote and to hold office. They shall be entitled to all the privileges of Associate members and upon the payment of $5.00 additional annual dues shall be entitled to Family privileges.

Section 5. Sustaining members shall be not less than twenty years of age. They shall pay annual dues of not less than $50.00. They shall be entitled to all of the privileges of Active members and shall have Family privileges as defined in Section 7 of this ARTICLE free of additional charge.

Section 6. Honorary members shall be entitled to all of the privileges of sustaining members, except the right to vote and to hold office. They shall be nominated by the Board of Managers and shall be elected by four-fifths of the votes of the members present at any stated meeting of the Institute at which their nomination may be acted upon.

Section 7. Family privileges consist of the right to receive a card for each individual in the family of the member and resident with the member entitling the holder to unlimited free admission to the Museum and the Planetarium.

Section 8. Members belonging to classes of membership existing prior to the amendment of this ARTICLE shall be reclassified in a manner consistent with their former dues and privileges.

ARTICLE III.

PAYMENT OF DUES.

Section 1. The annual fees for membership shall be due and payable on the first day of each month, whichever is nearest to the date of election or as determined by the Board of Managers.

Section 2. Any member whose dues are more than two months in arrears shall have all the privileges of membership suspended until such time as all arrears are paid. Should the dues not be paid when they become six months in arrears the said member shall forfeit his membership.

Section 3. The Board may remit temporarily in whole or in part the dues of any member either by action in a particular case or by establishing regulations governing certain cases.

Section 4. Every person admitted to membership in the Institute shall be considered as liable for the payment of dues until he shall have resigned, been dropped or have been relieved therefrom by the Board of Managers.

Section 5. Resignations of memberships shall be made to the Board of Managers in writing, but need not be accepted until all dues and arrears up to date of resignation shall have been paid.

Section 6. The privileges and title of Associate member may be obtained for life by paying therefor in one year the sum of $100. From this payment may be deducted one-half of the Student or Associate Membership dues paid by the member during the preceding ten years, but in no case shall the deduction exceed $50.00.

Section 7. The privileges and title of Active Member shall be enjoyed for life or may be obtained for life by a member who has heretofore or who shall hereafter pay therefor, in one year, the sum of $300, except that a person residing permanently at a distance of twenty-five miles or more from Philadelphia may become an Active Member for life by paying therefor in one year the sum of $100. From this payment may be deducted one-half of the dues paid by the member during the preceding ten years, but in no case

shall the deduction exceed one-half of said payment. Associate Members for life may become Active Members for life by paying therefor in one year the sum of $200.

Section 8. Firms, corporations, associations or individuals may nominate and subscribe for the membership dues of groups of members of any class or classes, at the annual dues provided for, subject to the approval of the Board as to any particular nominee. If the dues of these nominees amount to $100 or more in the aggregate, the firm, corporation, association or individual shall be known as an Affiliate of the Institute.

ARTICLE IV.

MANAGEMENT.

Section 1. The Institute shall be governed by a board of twenty-four (24) Managers elected by the members.

Section 2. The officers, who shall be elected by the Board of Managers, shall be a President, an Executive Vice President, not more than five Vice Presidents, a Secretary, an Assistant Secretary, a Treasurer, and an Assistant Treasurer. The Board of Managers may elect such other officers as it deems necessary.

Section 3. At the annual meeting of the Institute, eight Managers shall be elected each year to serve for three years, provided that the Managers now elected, or who may hereafter be elected, shall continue to serve until their successors be appointed.

Section 4. All elections of the Institute shall be by letter ballot and no vote may be cast by proxy.

Section 5. Nominations for Managers shall be made in writing at the stated meeting in the month of December. Each nomination paper must be signed by at least two members, who shall certify that the candidate will serve if elected. After the nominations are closed, the President shall appoint three members, who are neither officers nor nominees, to act as tellers of election. The list of nominees shall be posted at the Institute and incorporated (with directions for voting) in a ballot to be sent to each member by the Secretary at least one week before the date of election. Each ballot shall be accompanied by a return envelope addressed "To the Tellers of Election," and provided with a space for the signature of the member voting.

Section 6. On the date of the annual meeting, and at an hour previously designated by their chairman, the tellers shall meet at the Institute, and shall count all legal votes that have been received by mail or placed in the ballot box before eight o'clock P.M.; and when the count is completed they shall report to the annual meeting of the Institute the total number of ballots cast, together with the number of votes received by each candidate. Thereupon the presiding officer shall announce the names of the candidates who received the plurality of votes, and shall declare them elected Managers of the Institute for the ensuing terms.

Section 7. At the organization meeting of the Board of Managers, the Board of Managers shall elect the officers provided for in Section 2 of ARTICLE IV to serve for one year; and may at said organization meeting or from time to time thereafter elect such other officers as it may determine upon, and shall determine and fix the compensation, if any, to be paid to the officers so elected or appointed by them. The officers who are in office immediately prior to the annual meeting shall continue in office until their successors are elected or appointed by the Board of Managers as herein provided.

Section 8. Vacancies occurring in any office may be filled by the Board of Managers by election or appointment of persons to serve until the next annual election.

ARTICLE V.

BOARD OF MANAGERS.

Section 1. The Board of Managers shall have general charge and control of the Institute and of the Benjamin Franklin Memorial and The Franklin Institute Museum, and shall consist of twenty-four members elected as provided in ARTICLE IV. The President, the Executive Vice President, the Vice Presidents, the Secretary, the Treasurer, the Chairman of the Committee on Science and the Arts, and the Chairman of the Library Committee shall be *ex officio* members. A quorum of the Board of Managers shall be nine of the elected and *ex officio* members. The Board of Managers may adopt such by-

laws, rules and regulations for the governance of their affairs as are not inconsistent with the Charter and these By-Laws.

Section 2. They shall present, through the President, at the annual meeting of the Institute, a report of the condition of the affairs of the Institute.

Section 3. They shall hold stated meetings in each month except in July and August.

Section 4. Special meetings may be called by the President at his discretion and shall be called by him on written request of the Executive Committee or of any seven members of the Board. In case of his absence or refusal to act, such special meeting shall be called by the Secretary.

Section 5. Members who have not attended five regular meetings in the twelve months prior to the stated meeting of the Institute in December, shall be reported thereat as having resigned, unless it be unanimously voted by the Board at that meeting that such member has been absent for sufficient reason.

Section 6. All vacancies on the Board of Managers shall be filled by the Board until the next annual meeting of the Institute.

Section 7. The Board of Managers may elect an Executive Committee consisting of five of its Members with such authority to act on behalf of the Board as the Board may delegate.

ARTICLE VI.

DUTIES OF OFFICERS.

Section 1. The President shall be the executive head of the Institute and, under the supervision of the Board of Managers, shall have general charge of the affairs of the Institute. He shall preside at all meetings of the Institute and of the Board of Managers and shall be, *ex officio*, a member of all standing committees of the Institute.

Section 2. The Executive Vice President, under the supervision of the President, shall have immediate charge of all affairs of the Institute other than those matters specifically delegated by action of the Board of Managers pursuant to these By-Laws to the Executive Committee and the Finance Committee. He shall prepare the annual budget, and submit it to the Board of Managers for approval. He shall be an *ex officio* member of all standing Committees of the Institute. He shall report to the President. In case of the disability of the Executive Vice President, the President shall designate another officer or other officers to assume his duties.

Section 3. In the absence of the President, the Executive Vice President shall exercise his duties. In the absence of both the President and the Executive Vice President, the Vice Presidents shall exercise the duties of President in order of their seniority in office.

Section 4. The Secretary shall be responsible for keeping the minutes of all meetings of the Institute and of the Board of Managers, shall keep the records of the Institute, and shall perform all the duties usually pertaining to the office of Secretary. He shall report to the Executive Vice President. In the absence or disability of the Secretary, the Assistant Secretary shall perform his duties.

Section 5. The Treasurer shall have custody of all monies received from the committee on Finance together with all monies received by the Institute from dues, admissions, and other operations of the Institute, depositing them, in the name of the Institute, in such bank or banks as the Board of Managers shall direct and disbursing them by checks signed as the Board of Managers shall likewise direct. He shall keep accounts of the receipts and disbursements and shall report thereon to the Board of Managers as directed. He shall have general supervision of the accounts of the Institute and shall render such financial statements as directed by the Board of Managers. He shall give bond to an amount and with such surety as the Board of Managers shall determine. The Treasurer shall report to the Executive Vice President. In the absence of the Treasurer, the Assistant Treasurers shall, in the order of their seniority in office, perform his duties.

ARTICLE VII.

BOARD OF COUNCILLORS.

The Board of Councillors shall consist of not more than fifty members and shall embrace, as *ex officio* members, the Governor of Pennsylvania, the Chief Justice of the Supreme Court of Pennsylvania, the Mayor of the City of Philadelphia, the President of the City Council of Philadelphia, the President of the Board of Public Education and the

Superintendent of Schools of the City of Philadelphia, the Provost and the President of the University of Pennsylvania, the President of Commissioners of Fairmount Park, the President of the American Philosophical Society, the President of the Poor Richard Club, and the President and Secretary of The Franklin Institute, which two latter shall respectively be Chairman and Secretary of the Board of Councillors. Additional members shall be nominated by the President or Secretary of the Institute and elected by the Board of Managers, at such intervals as the Board of Managers may see fit. The Board of Councillors shall meet at the call of its Chairman or of the Board of Managers for the consideration of any matter that pertains to the welfare of the Institute. The Secretary shall from time to time send to the members of the Board of Councillors a report of the activities and accomplishments of the Institute.

ARTICLE VIII.

COMMITTEES OF THE INSTITUTE.

Section 1. There shall be the following Standing Committees of the Institute:

1. Bartol Research Foundation Committee.
2. Biochemical Research Foundation Committee.
3. Committee on Science and the Arts.
4. Endowment Committee.
5. Finance Committee.
6. Library Committee.
7. Meetings Committee.
8. Membership Committee.
9. Museum and Memorial Committee.
10. Publications Committee.
11. Committee on Research.

Section 2. Each standing committee of the Institute shall consist of the number of members determined on by the Board of Managers and composed of members of the Board of Managers and/or non-members as may be desirable, except that the Committee on Science and the Arts shall consist of not less than sixty nor more than seventy-five members, and except that the Bartol Research Foundation shall consist of not less than ten nor more than fifteen members as provided for in Section 7 of this ARTICLE.

Section 3. The members of each committee shall serve one year, except that members of the Committee on Science and the Arts shall serve three years.

Section 4. The members of each committee, other than *ex officio* members specifically provided for by these By-Laws, shall be appointed by the President and approved by the Board of Managers at the organization meeting of the Board following the annual meeting except that each year but one-third of the total number of members of the Committee on Science and the Arts shall be so appointed and approved.

Section 5. The Chairman of each Committee shall be designated by the President, except that the Library Committee and the Committee on Science and the Arts shall select their chairmen. A chairman who is not an elected or *ex officio* member of the Board of Managers shall have the privilege of the floor at meetings of the Board but shall not have the right to vote.

Section 6. Each Committee shall report monthly to the Board of Managers through its Chairman. A copy of their report shall be in the hands of the Executive Vice President one week before the monthly meeting of the Board.

Section 7. The Bartol Research Foundation Committee shall consist of not less than ten nor more than fifteen members, including the President and the Executive Vice President of the Institute and the Director of the Bartol Research Foundation as *ex officio* members and nine members of the Institute of whom not more than four shall be members of the Board of Managers. The Committee shall, after consultation with the Director of the Bartol Research Foundation and subject to the approval of the Board of Managers, collaborate with the Executive Vice President in carrying out the purposes and determining the policies of the Bartol Research Foundation. It shall appoint the Director and the Staff of the Laboratories, subject to the approval of the Board of Managers. The Director of the Bartol Research Foundation, to whom all other employees of the Foundation shall report, shall have charge of the scientific activities of the Foundation and shall report on all matters to the Committee or to the Executive Vice President, as the Committee may elect.

Section 8. The Biochemical Research Foundation Committee shall report to the Board of Managers from time to time on the condition of the affairs and operation of said Foundation and shall be available to the Director of the Biochemical Research Foundation and its Advisory Council for collaboration and advice.

Section 9. The Committee on Science and the Arts shall investigate current discoveries, inventions and other achievements in the sciences and their application in the mechanical and industrial arts with a view of affording such recognition as may lie within the power of the Institute to bestow. The Committee shall exercise independent judgment therein. In all matters of a general or administrative nature the Committee shall collaborate with the Executive Vice President.

Section 10. The Endowment Committee shall, in collaboration with the Executive Vice President, encourage the gift of funds to the Institute.

Section 11. The Finance Committee shall have the custody and control of all the securities and investments of the Institute with full power and authority to buy and to sell, and to invest and reinvest the same; including the power to satisfy mortgages and extinguish ground rents, and to direct the placing of all such insurances on Real Estate held for investment as it may deem necessary, and to make such improvements, repairs and alterations to such Real Estate as it may deem necessary. It shall have power to authorize the proper officers of the Institute to execute the necessary papers to effect all purchases, sales and assignments of property other than real estate; to transfer registered securities; to subscribe to bond-holders' agreements to plans of reorganization involving any securities held by the Institute or in which it has an interest; and to do all such acts as are necessary in pursuance of the foregoing powers. It may, with the approval of the Board of Managers, appoint a Trust Company of the City of Philadelphia to act as Fiscal Agent under the direction of the Committee.

The Finance Committee shall keep a record of all its acts and proceedings, which shall be communicated to the Board of Managers.

Section 12. The Library Committee, in collaboration with the Executive Vice President, shall be of counsel and advice in the purchase of books and publications suitable to and consonant with the purposes of the Library; and in other matters pertaining to the Library.

Section 13. The Meetings Committee shall, in collaboration with the Executive Vice President, secure for presentation before the Institute papers dealing authoritatively with subjects of import in the fields of science, engineering and industry.

Section 14. The Membership Committee shall, in collaboration with the Executive Vice President, promote the several memberships defined in ARTICLE II of these By-Laws.

Section 15. The Museum and Memorial Committee shall, in collaboration with the Executive Vice President, be of counsel and advice to the Director of the Benjamin Franklin Memorial and The Franklin Institute Museum, and with the scientific staff of the Museum concerning the activities and the exhibits of both the Museum and the Memorial.

Section 16. The Publications Committee, in collaboration with the Executive Vice President, shall be of counsel and advice with regard to all publications of the Institute other than publications of the Bartol Research Foundation.

Section 17. The Committee on Research shall, in collaboration with the Executive Vice President, be of counsel and advice to all research activities of the Institute except those of the Bartol and Biochemical Foundations.

Section 18. Each committee shall organize and adopt rules as it sees fit, subject to the provisions of these By-Laws and the approval of the Board of Managers.

ARTICLE IX.

MEETING.

Section 1. The Institute shall hold stated meetings on the third Wednesday of each month, except in June, July, August and September, at 8:15 P.M.

Section 2. Special meetings shall be called by the President upon the order of the Board

of Managers or within ten days upon the written application of twenty-four voting members of the Institute.

Section 3. Twenty-four members shall constitute a quorum at any stated or special meeting.

Section 4. The Annual Meeting of the Institute shall be held on the third Wednesday of January of each year at 8:15 P.M. Elections at Annual Meetings shall not result from the poll of less than forty-eight ballots cast in accordance with ARTICLE VI, Section 6, of these By-Laws.

ARTICLE X.

AMENDMENTS.

These By-Laws may be altered or amended at any stated meeting of the members of the Institute, provided notice in writing, signed by two members, of the proposed alteration or amendment, shall be given to the Board of Managers two months prior to the said meeting, except that amendments to ARTICLE I, relating to capital stock, must be ratified subsequently by a majority of the stock represented at a meeting specially called for this purpose.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, January 14, 1948.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JANUARY 14, 1948.

MR. WALTER C. WAGNER in the Chair.

The following report was presented for final action:

No. 3187 } Franklin Medal.
No. 3188 }

This report recommended the award of two Franklin Medals; one to Wendell Meredith Stanley, of Princeton, New Jersey, "In recognition of his discovery that a virus can be a nucleo-protein rather than a living organism, thereby establishing a significant turning point in virus research; and also of his development of an effective centrifuge-type influenza vaccine by means of which the intensity of future influenza epidemics may be greatly lessened"; and one to

Theodor von Kármán, of Pasadena, California, "In recognition of his outstanding engineering and mathematical achievements, particularly those relating to the development of advanced aerodynamic conceptions which have directly influenced the progress of aeronautical design, and for his unusual leadership whereby some measure of his own genius is constantly instilled in those who work with him."

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents plus ten cents for each order and postage.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M., Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS.

AERONAUTICS.

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SPORN, P. AMBROSE, AND T. BAUMEISTER. Heat Pumps. 1947.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

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HOUGEN, O. A., AND K. M. WATSON. Chemical Process Principles. Volume 2. 1947.

KAUSCH, OSKAR. Handbuch der Künstlichen Plastischen Massen. 1939.

MARKLEY, KLARE S. Fatty Acids, Their Chemistry and Physical Properties. 1947.

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NORD, F. F., EDITOR. Advances in Enzymology. Volume 7. 1947.

PATRICK, W. L., AND OTHERS. German Glass Industry. 1947.

PHILLIPS, F. C. An Introduction to Crystallography. 1946.

REILLY, J., AND W. N. RAE. Physico-Chemical Methods. Volumes 1 and 2. Fourth Edition. 1943.

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KRAEHENBUEL, J. O., AND M. A. FAUCETT. Circuits and Machines in Electrical Engineering. 1947.

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HOLDEN, P., L. FISH AND H. SMITH. Top Management Organization and Control. 1947.

LIPPERT, FREDERICK G. Accident Prevention Administration. 1947.

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KEYSER, CASSIUS J. Mathematics as a Cultural Clue. 1947.

MEYER ZURCAPELLEN, W. Mathematische Instrumente. 1944.

WALD, ABRAHAM. Sequential Analysis. 1947.

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NEWTON, JOSEPH. An Introduction to Metallurgy. Second Edition. 1947.

MILITARY ENGINEERING.

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ARLEY, NIELS HENRIK. On the Theory of Stochastic Processes. 1943.
BOUWERS, ALBERT. Achievement in Optics. 1946.

PHOTOGRAPHY.

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ADDISON, HERBERT. Hydraulic Measurements. 1946.
EINSTEIN, ALBERT. Relativity; The Special and General Theory. 1947.
FRISCH, O. R. Meet the Atoms. 1947.
MINORSKY, N. Non Linear Mechanics. 1946.
NAHMIAS, MAURICE E. Artillerie Atomique. 1947.

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SHEA, J. G., AND WEGNER, P. N. Woodworking for Everybody. 1946.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Elegy Written in (and for) a City Laboratory Shortly before Moving to the Country

(addressed to Dr. McDonald in recognition of the seventh anniversary of the removal of the laboratories of the Biochemical Research Foundation from temporary quarters in the loft and in parts of the 2nd, 4th, and 5th floors of the Integrity Trust Building, 133 South 36th Street, Philadelphia, to permanent quarters in Newark, Delaware, on the third Monday in December, 1940).

The clock now marks the hour when tasks may cease.
Experiments draw quickly to a close.
South new commuters speed by car and train
And leave the Lab to rabbits and repose.

Now twinkle distant buildings through the dusk,
And all the Lab with stillness is replete,
Save where the cockroach weaves his hurried flight
And muffled honks drift upward from the street.

Save that on yonder whirring thermostat
The regulator lamps switch off and on,
And the night watchman, flashlight in his hand,
Makes his accustomed, solitary round.

Beneath these rugged beams, these skylights too,
Where rooms in labyrinthine maze extend,
Each in his transitory bit of space,
Soon o'er their tasks no more the Staff shall bend.

The supersonic oscillator's screech,
The signal system with its raucous calls,
The din of generator and of lathe
No longer shall resound within these walls.

Here soon no more the blazing still shall burn,
Or conf'rence members ply their busy wits,
Famed visitors address the Seminar,
Nor ultracentrifuges fly to bits.

The books, the benches, all the apparatus
That Allen, Schroeder, Ely e'er okayed
Await alike the inevitable hour
With little gadgets Dietz and Kelley made.

Full many a fact of once potential worth
Shall with its journal be discarded too.
Full many a reprint's doomed to burn unread,
And waste its data on the chimney flue.

Far from the madding crowd's ignoble strife
A fair new structure nears its opening day.
There, amid sylvan groves and verdant fields,
We'll keep the noiseless tenor of our way.

Yet e'en these paper walls, this narrow stair,
These few dark rooms which never see the sky,
This humble setting for equipment rare,
Implore the passing tribute of a sigh.

For who, to dumb forgetfulness a prey,
The meanest habitation e'er resigned,
Left the fond scenes of earnest hours of toil,
Nor cast one longing, ling'ring look behind?

From some one man each institution stems,
In some one soul first glowed the vision clear.
Remembering this Lab, remember too
The one whose vision gathered substance here.

Haply some one among you may exclaim:
"Oft have we seen him at the close of day
Gaze from this window till the town lights shone,
Wondering, 'Can I meet the next payday?'

"Broad was his planning, and his purpose clear.
All that he lacked was ample funds to spend.
He gave to setbacks (all he owed) a sneer
And gained from heaven (more than he'd hoped) a friend."

L. K.

BOOK REVIEWS

THE TECHNOLOGY OF ADHESIVES, by John Delmonte. 516 pages, drawings, illustrations, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1947. Price $8.00.

The importance of adhesives is increasing rapidly, with a strong trend toward the synthetic resin developments. This is seen in their applications which were stimulated a great deal by World War II. These include a number of new large scale applications of boat, aircraft and building construction. And aside from the wood field, adhesives for metals show much promise. The unity of aluminum alloys and the joining of steel with joint shear strengths of 3000 p.s.i. are an accomplished fact. Ceramics, glass, textiles, rubbers, and solid organic plastics are being bonded with adhesives which form glue lines stronger than the materials themselves. Confidence has been inspired in designers and engineers in the permanence of the bonding quality. There is much to be done in further development of adhesives that may revolutionize fabrication processes. This book is a treatise on the subject in its present state of knowledge.

There are twenty chapters in all, covering the many types of adhesives including phenolic resin, urea and melamine, polyvinyl resin, polystyrene and acrylic resin, rubber, cellulose derivatives, protein substances, vegetable and animal glues and sodium silicate. A good foundation is laid in the introduction to the work where chemical and physical classifications of adhesives are shown and considerable part of the book is devoted to theories of adhesive action. They are grouped here also as chemical and physical, but it is pointed out that there is much basic information still necessary to explain many unusual phenomena which arise in establishing an adhesive bond. Following this, attention is directed to adhesives for various applications such as the joining of wood, organic plastics, metal and rubber, tapes, papers, clothes, foils and for inorganic materials such as chemical stoneware, mica plate, glass, etc. The last chapter is on tests and specifications, reference being made to the specifications developed at Wright Field, Forest Products Laboratory, Naval Aircraft Factory, and the Bureau of Standards during the war. While most of this work related to wood, some referred to the bonding of metal parts. A review is given of other sources. There is a subject index in the back of the book adding greatly to its use as a reference.

The book is an up-to-date treatment of adhesives, their many kinds, compositions, uses, with theories, chemical and physical principles and characteristices. Where limited knowledge or differences of opinion exist it is so stated. The book gives an excellent exposition presented in a logical manner and arranged for ready reference.

R. H. OPPERMANN.

TUNGSTEN, K. C. Li and Chung Yu Wang. Second edition. 430 pages, tables and illustrations, 15 × 24 cms. American Chemical Society Monograph No. 94. New York, Reinhold Publishing Corporation, 1947. Price $8.50.

The special characteristics of tungsten for heat resistance and acid proofing have made tungsten a very important metal, both in time of war and in time of peace. During the last quarter century, no other metal in such comparatively small quantity has had so profound an influence upon the conduct and development of industries as has tungsten. In spite of the discovery of its various new uses during recent years, the future of tungsten still holds much promise. The book at hand, an American Chemical Society Monograph, is a treatment of the various aspects of the subject giving the state of present knowledge of it.

After a brief reference to the history of tungsten, the geology of tungsten is taken up including reference to deposits in various countries in the world, and this is followed by a description of principles and processes adopted for ore dressing. A section devoted to the metallurgy of tungsten points out that this is largely empirical and is based mainly on patents. The treatment here is divided into subheadings of decomposition of tungsten ore, purification of

tungstic oxide, and production of tungsten powder. The physical and chemical properties of tungsten are next given. Attention is given to the compounds of tungsten and oxygen. The progression then is through analysis of tungsten, the industrial application of tungsten, substitution of tungsten and the economics of tungsten. Appendices include the terms of purchase of tungsten, ores, Chinese contracts, the post-war tungsten situation and references relating to tungsten alloys. In the back there is an extensive subject index.

The work is a combination of digested literature and expert knowledge of the subject. It is well divided and arranged in logical order, easy to read, and readily adaptable as a reference with many helpful and useful tables, diagrams, curves, drawings and illustrations. It should stimulate interest of those who desire to know the present state of knowledge of the subject, as well as future possibilities.

R. H. OPPERMANN.

FUNCTIONAL ANATOMY OF THE MAMMAL, by W. James Leach. First Edition. 231 pages, illustrations, 15 × 24 cms. New York, McGraw-Hill Book Co., Inc., 1946. Price $2.50.

For various reasons it is practical, useful and necessary to use bodies of animals when studying anatomy. Many students properly ask why the body of the domestic cat is so commonly studied rather than some other type of mammal. The reasons of size and availability are obvious but from the biological standpoint the cat may be used as a fairly representative type, occupying a position near the middle of the list of classification of mammals. From the purely structural standpoint, man is not far removed from the position of the cat. Terminology and methods learned in the dissection of the cat are readily applicable to the human body.

A large part of the material in this book deals with the structure of the common domestic cat. The book is designed to make anatomical terms really intelligible by integrating a rather specific laboratory study of the cat with text material of a more general character with emphasis on man. Functional organization is stressed rather than how each of the parts is constructed as a separate unit. After a chapter on general considerations which includes a list of terms and definitions and sufficient background information, there is discussed the skeletal system, superficial dissection of the cat, a study of the voluntary muscles and a survey of the internal anatomy. Following this, there are sections devoted to the alimentary system, the respiratory system, the vascular system, the urogenital system and the nervous system. The latter part of the book is devoted to special sensory apparatus and the endocrine organs. An appendix gives a method of laboratory preparation and preservation of materials which is unusual. A subject index is in the back.

The presentation is clear and readily adaptable to a laboratory study. It meets its designed purpose satisfactorily and should be of value to medical students as well as students in experimental physiology, pharmacology and veterinary practice.

R. H. OPPERMANN.

GERMAN RESEARCH IN WORLD WAR II, by Leslie E. Simon. 218 pages, drawings and illustrations, 15 × 24 cms. New York, John Wiley & Sons, Inc, 1947. Price $4.00.

Those who have followed the changes in arms during the recent war have frequently seen new arms brought to the battlefield by the Germans and have wondered through just what channels the main ideas passed before they became actualities. How extensive was their research and to what extent was it especially organized for war work? How did they define basic research, technical research, engineering design, development, and production? Where were the weak spots in their system?

The author of this book, Colonel Leslie E. Simon, U. S. Army, was sent to Europe to go to German scientific establishments as soon as possible after their capture, examine what evidence there was and question personnel, in an effort to determine the structure and functioning of German research. Generalizations of this information and case histories which are permissible from the viewpoint of military security are given in this book. In presenting his story Colonel Simon gives considerable background as reasons for situations. Thus in the beginning he tells of various factors or influences which had important effects on research such as quantity and

quality of apparatus and equipment, character of research staff, degree of independence of research and extent of non-scientific direction. Also an adherence to a defined and consistent terminology has been necessary, descriptions of laboratories and concepts of psychological influences, and general conditions which prevailed in the German war ministries before the war. With this as preparation, a description of the German research organization and its functioning, a version of its accomplished research, and a basis for analysis and criticism of the whole pattern of the German scientific effort are presented.

The value of this work lies in the fact that it can be regarded as a study of a completed laboratory experiment, pointing out the inabilities of the military to evaluate and promote or reject the projects of civilian scientists, as well as the successes which were achieved. A wide variety of subjects is covered including ballistics, aerodynamics, instruments and measurement techniques, the wind gun, sound as a weapon, and explosive powered vortices. The book has much in it for the non-technical as well as the technical man interested in accomplishments and methods of organized research.

R. H. OPPERMANN.

PHYSICAL CONSTANTS OF HYDROCARBONS, by Gustav Egloff. Volume IV. American Chemical Society Monograph Series. New York, Reinhold Publishing Corporation, 1947. Price $17.50.

This well known series of books has been added to by the present Volume IV which is actually a continuation of Volume III, and an evaluation of the physical constants of polynuclear aromatic hydrocarbons. The compounds included under this classification are fused ring hydrocarbons having at least one aromatic ring. The nomenclature used is mainly the "Ring Index" but complications arising in the nomenclature of many derivative compounds necessitated departures from this system. It is chosen with the intention of offering pictures of the structure which may be readily visualized. The order of tabulation also offered a problem which has been admirably solved. All compounds having a common empirical formula are classified in one group. These groups are numbered with Roman numerals and listed in order of decreasing hydrogen content. Except for sections entitled miscellaneous and a few sections in which general classification is simplified by listing together such similar compounds as anthracene and phenanthrene derivatives, the compounds of one section have a common nucleus.

It is stated in the work that although thousands of polynuclear aromatic hydrocarbons have been prepared, these compounds represent only a minor fraction of the theoretical possibilities. A great number of the larger polynuclears have been synthesized in recent years, particularly in connection with cancer research and other physiological problems. The present study also shows that the published data for many of the known compounds are insufficient for the identification of these compounds on the basis of properties alone.

This American Chemical Society Monograph is a useful addition to an already well known series.

R. H. OPPERMANN.

PATENT NOTES FOR ENGINEERS, by C. D. Tuska. 165 pages, illustrations, 15 × 23 cms. Princeton, N. J., RCA Review, 1947. Price $2.50.

With this volume the RCA Review Department of RCA Laboratories Division introduces a new *Engineering Book Series*, to include subjects of general engineering interest which do not fit into their various other publications.

Mr. Tuska, Director of the Patent Department of the RCA Laboratories Division, has prepared these notes on patents aided by other members of the department. The stated purpose of the volume is "to bridge the technical gap between engineers, research workers and inventors generally, and their patent attorneys." To achieve this end the author has pointed out the need for proper protection of an invention by patenting, even for the inventor who, wishing to dedicate his invention to the public good, must patent it to prevent someone else from doing so and securing its benefits.

Since there is no definition of invention in either the United States Constitution or the Federal Statutes, there is presented a lengthy discussion of statutory invention as defined negatively by the courts. Specific cases are presented with citations from the court decisions. However even these "negative rules" have their exceptions which are pointed out. Warning is given that not all the "negative rules of invention" have been presented and that particular cases may well have to be decided by the Patent Office and the courts as to the presence or absence of "patentable invention." Various essentials of statutory invention are enumerated and discussed with examples.

The succeeding chapters consider certain definite points of great importance in the protection of inventions and for which the inventor is responsible. These are the disclosure of the invention, diligence in reduction to practice and the securing of witnesses to both the disclosure and the successful demonstration of the invention. In the discussion of the prosecution of patent applications and the handling of interferences, if such arise, the importance of these precautionary measures is amply demonstrated with adequate citation of cases. A concluding chapter considers the matter of patent ownership and use, especially as relating to employee and employer.

This is not a manual of procedure for the inventor as relating to patents, although it offers many practical suggestions, but rather a discussion of the legal aspects back of those suggestions. Such a presentation should make the inventor well aware of the need for taking the necessary precautions to adequately protect his invention.

G. E. PETTENGILL.

WIND-TUNNEL TESTING, by Alan Pope. 319 pages, illustrations, 14 × 21 cms. New York, John Wiley & Sons, Inc., 1947. Price $5.00.

In the development of new airplanes wind-tunnels play a vital role. Although there have been many papers on the various components of the tunnel and on corrections to be applied to the data secured, there has been no one comprehensive work on wind-tunnels. The present volume aims to fill this gap in aeronautical literature.

The many types of wind-tunnels are reviewed briefly. An accompanying list gives data respecting more than one hundred tunnels throughout the world, the listing being as complete as security restrictions permitted.

In attacking the problem of design, the author limits himself to the general utility tunnel. His treatment of the propellor-flow straightener system is precise but complete. The determination of the flow characteristics of the tunnel after construction are detailed. These include angular-flow variation in the jet, velocity variation in the jet, longitudinal static-pressure gradient and turbulence.

The various load measurements of the model that are required are noted with a detailed description of the different types of balances employed and nine different kinds of mountings. The actual testing procedure is discussed in full with sections on the various components of the plane as well as the complete model.

The important factor of wind-tunnel-boundary corrections receives full attention with downwash corrections discussed for various types of jets. The difficult problem of extrapolation to full scale is also considered. Auxiliary testing equipment, the use of small wind-tunnels and notes on wind-tunnel-model construction are additional material treated.

Numerous worked examples, suggested problems and ample references add to the value of the book for the student and the engineer.

G. E. PETTENGILL.

VAT DYESTUFFS AND VAT DYEING, by M. R. Fox. 323 pages, illustrations, 14 × 21 cms. New York, John Wiley & Sons, Inc., 1947. Price $5.50.

With the rapid growth of the synthetic dyestuffs and rayon industries, the whole art of dyeing has become exceedingly complicated. The present volume is an attempt to deal in detail with one aspect of the subject—vat dyestuffs and their application.

The book is essentially a practical one. After two introductory chapters dealing with the indigo and related dyes and the anthraquinonoid vat dyestuffs, the author discusses different dyeing methods. Agents to be used and the properties of the dyestuffs are considered as well as various fastness tests.

The application to animal fibers and cellulosic fibers is fully treated with indication of which dyes to use with which materials. The application of the indigosol dyestuffs is treated separately. Although the author does not pretend to offer a detailed account of textile printing, the chapter devoted to it is the longest in the volume. The descriptions and illustrations of machinery should prove of value. Non-textile uses of vat dyestuffs are touched on briefly, and there is a chapter on the methods of identification of vat dyestuffs. A useful feature is the extensive listing of commercial vat dyestuffs grouped into equivalent, or analogous, products.

The entire work reveals the author's practical knowledge of the subject and with the many examples should be a valuable text for anyone working in the field—student or practicing dyer.

G. E. PETTENGILL.

THE STRANGE STORY OF THE QUANTUM, An Account for the General Reader of the Growth of the Ideas Underlying Our Present Atomic Knowledge, by Banesh Hoffmann. 239 pages, illustrations, 14 × 20½ cms. New York, Harper & Brothers, 1947. Price $3.00.

In his telling of the complex story of the quantum theory, Dr. Hoffmann has used a wealth of analogies and similes with which to stimulate the interest of the reader. Presented as a struggle between the wave and the particle theories, the account often becomes involved and these artificial devices of the author prove helpful to the layman.

The book does not aim to present the mathematics involved, but only to serve as a "guide to those who would explore the theories by which the scientist seeks to comprehend the mysterious world of the atom." And as the author warns the story of events and discoveries is indeed complex, with many outlandish ideas, some good and some bad. The quantum theory was indeed the beginning of a new era. The persons mentioned run the gamut of those who have played a part in fitting together the fragments of the present picture. The first investigators Planck, Einstein, Bohr are followed by De Broglie, Heisenberg, Dirac and Schrodinger among others. There is no need to repeat here all that the volume contains but if you like your science served up with a dash of spice, humor and satire you will enjoy this discussion of the development of the quantum theory.

G. E. PETTENGILL.

THE CHEMISTRY AND TECHNOLOGY OF WAXES, by Albin H. Warth. 519 pages, drawings, tables and illustrations, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1947. Price $10.00.

The English term *wax* is derived from the Anglo-Saxon *weax*, which was the name applied to the natural material of the honeycomb of the bee. When a material of similar nature was found in plants it also became known as *weax* or *wachs*, and later wax.

In modern times the term wax has taken on a broader meaning and is applied to all wax-like solids and liquids found in nature, and to those which occur individually in waxes, such as hydrocarbons, acids, alcohols and esters.

The literature on the subject of waxes is abundant, but widely scattered. In this volume the author brings together and correlates much material that is not available to one lacking the facilities of an extensive library.

After a brief discussion on the chemistry of waxes, the various subdivisions are dealt with. These include the natural waxes, fossil and earth waxes, lignite paraffins, petroleum waxes, synthetic waxes and wax compounds. Chapters on emulsifiable waxes, methods for determination of constants, and on wax technology conclude the work.

An appendix containing tables of physical constants of waxes is attached.

HENRY N. MICHAEL.

THE RARE-EARTH ELEMENTS AND THEIR COMPOUNDS, by Don M. Yost, Horace Russell, Jr., and Clifford S. Garner. 92 pages, drawings and tables. New York, John Wiley & Sons, Inc., 1947. Price $2.50.

Including lanthanum, fifteen chemically similar elements between barium and hafnium on the periodic table are known as rare earth elements. This region has been the least investigated by the spectroscopists. The book at hand presents the principal chemical and physical properties of the rare earth elements and their compounds, and shows the concordance of the current theories with these properties. The six chapter headings are indicative of the coverage. They are Electronic Structures and Oxidation States of the Rare Earth Elements, Paramagnetic Properties of Rare Earth Compounds, Absorption Spectra of Rare Earth Compounds, Evidence For the Existance of Element 61, Separation of Rare Earths, and Chemical and Physical Properties of the Rare Earths. The appendices are Nuclear Properties of the Rare Earth Elements, General Physical Constants, and The Periodic System of the Elements. Many references are made to the literature. In the back are name and subject indexes.

This 86-page treatment contains a fund of information digested from various sources and coupled with expert knowledge of the authors.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Alexander Dallas Bache, Scientist and Educator, 1806–1867, by Merle M. Odgers. 223 pages, 14 × 21 cms. Philadelphia: University of Pennsylvania Press, 1947. Price $2.75.

Improvising Supervision, by Frank Cushman and Robert W. Cushman. 232 pages, 13 × 19 cms. New York: John Wiley & Sons, Inc., 1947. Price $2.50.

Theory of Limit Design, by J. A. Van Den Broek. 144 pages, drawings, 14 × 22 cms. New York: John Wiley & Sons, Inc., 1947. Price $3.50.

Energy Unlimited, The Electron and Atom in Everyday Life, by Harry M. Davis. 273 pages, illustrations, 14 × 22 cms. New York: Murray Hill Books, Inc., 1947. Price $4.00.

Techniques of Observing the Weather, by B. C. Haynes. 272 pages, tables, drawings and illustrations, 14 × 22 cms. New York: John Wiley & Sons, Inc., 1947. Price $4.00.

Photographic Facts and Formulas, by E. J. Wall and Franklin I. Jordan. 364 pages, drawings and tables, 16 × 24 cms. Boston: American Photographic Publishing Co., 1947. Price $5.00.

Engineering Applications of Fluid Mechanics, by J. C. Hunsaker and B. G. Rightmire. 494 pages, drawings and illustrations, 15 × 24 cms. New York: McGraw-Hill Book Co., 1947. Price $5.00.

Composition and Pictures, by Eleanor Parke Custis. 224 pages, illustrations, 22 × 29 cms. Boston: American Photographic Publishing Co., 1947. Price $6.00.

Electronic Circuits and Tubes, by Electronics Training Staff of the Cruft Laboratory. 948 pages, drawings, 15 × 23 cms. New York: McGraw-Hill Book Co., 1947. Price $7.50.

The Scientific Paper, How to Prepare It, How to Write It, by Sam F. Trelease. 152 pages, 13 × 19 cms. Baltimore: The Williams & Wilkins Co., 1947. Price $2.00.

Understanding Vectors and Phase, by John F. Rider and Seymour D. Uslan. 153 pages, drawings, 13 × 19 cms. New York: John F. Rider Publisher, Inc., 1947. Price $.99 (paper).

Introduction to Modern Physics, by F. K. Richtmyer and E. H. Kennard. Fourth Edition. 759 pages, drawings, tables and illustrations, 16 × 23 cms. New York: McGraw-Hill Book Co., 1947. Price $6.00.

Organic Syntheses, by R. L. Shriner, Editor. 121 pages, 15 × 24 cms. New York: John Wiley & Sons, Inc., 1947. Price $2.25.

Fundamental Electronics and Vacuum Tubes, by Arthur Lemuel Albert. Revised Edition. 510 pages, drawings and illustrations, 16 × 24 cms. New York: The Macmillan Company, 1947. Price $6.00.

Fundamentals in Chemical Process Calculations, by Otto L. Kowalke. 158 pages, tables and drawings, 15 × 22 cms. New York: The Macmillan Company, 1947. Price $2.80.

High Frequency Measuring Techniques Using Transmission Lines, by E. N. Phillips, W. G. Sterns, N. J. Gamara. 58 pages, diagrams, 21 × 28 cms. New York: John F. Rider Publisher, Inc., 1947.

CURRENT TOPICS.

Expanding Air Causes Sudden Temperature Drop.—A bottle of "pop" from the refrigerator, or a tiny balloon such as Junior makes by sucking on a broken piece of rubber, may have little to do with the weather, but each has been found capable of causing a snowstorm, the General Electric Company reported recently.

Or digging still deeper into Junior's treasure trove, that popgun he got for Christmas also can "make it snow," the company said.

According to Dr. Bernard Vonnegut, scientist of the General Electric Research Laboratory, supercooled water droplets in a cold-chamber cloud can be transfromed to snow simply by uncapping the bottle of "pop," bursting the toy balloon or shooting off the popgun, in the cloud.

In a similar experiment out-of-doors, Dr. Vonnegut reported he has been able to change supercooled fog in his backyard into snow by firing a popgun.

Although seemingly unscientific, the new methods for producing man-made snow each employ the same fundamental principle that sudden expansion of air will cause a severe drop in temperature of the air expanded.

"In each case, the temperature drops low enough to produce spontaneously millions of ice nuclei," Dr. Vonnegut said. "If these nuclei, as they are created, are dispersed into a supercooled cloud, or a cloud whose water droplets are liquid even though below freezing in temperature, the ice nuclei will grow at the expense of the water droplets, and snow will result."

The sudden drop in temperature of the expanded air, taking place in approximately 1/1,000th of a second, must go down at least to minus 31 degrees Fahrenheit, according to Dr. Vannegut, since previous experimentation has proved that ice nuclei form spontaneously only at that temperature or lower.

In tests with a toy balloon, Dr. Vonnegut desired an extremely small balloon, and thus he resorted to the favorite balloon trick of children, in which a tiny bit of rubber becomes inflated by sucking it.

A balloon about the size of a BB shot produced approximately 100,000,000 nuclei when burst into a supercooled cloud, Dr. Vonnegut estimated.

Experiments with a bottle of "pop," in which the air in the neck of the bottle was expanded by removing the cap, furnished an answer to the common mystery of why a bottle of liquid, kept in a refrigerator a considerable time, sometimes doesn't freeze until opened.

"The liquid in the bottle becomes supercooled in these instances," Dr. Vonnegut said, "and when the bottle is opened, the sudden temperature drop caused by expansion of air in the neck makes ice nuclei form. The nuclei then cause the liquid to change to ice."

Dr. Vonnegut said that an automobile tire blowout or the unhooking of couplings on a railroad car might produce snow artificially if atmospheric conditions were right. "If a tire blew out or a coupling were unhooked in a condition of supercooled fog, the sudden momentary drop in temperature caused by air expansion would produce ice nuclei, which in turn would seed the supercooled fog and create snow crystals."

The technique of expanding air to cause a sudden temperature drop constitutes a third means for producing snow investigated under a current snow research program being conducted by the General Electric Company in conjunction with the U. S. Army Signal Corps Engineering Laboratories at Bradley Beach, N. J. and the Army and Navy air forces.

Other means developed for making snow include scientist Vincent J. Schaefer's dry-ice technique, in which the seeding of a cloud with dry-ice causes ice nuclei to appear resulting in formation of snow, and the foreign-nuclei technique, in which certain materials, foreign to snow but resembling it in crystalline structure, actually "fool" water particles in a supercooled cloud by serving as nuclei and growing into snow crystals at the expense of the water particles.

R. H. O.

More Sugar from Graded Fields.—"Turtlebacking," as a descriptive term, gives a good idea of a new method of handling the drainage problem in some of the low lying and flat sugarcane fields in Louisiana. On some soils, at least, the U. S. Department of Agriculture finds the increased sugar yield from a single crop more than pays the expense of "turtle backing." The Soil Conservation Service is investigating the possibilities on other types of soil.

In the Sugar Bowl country, drainage is a No. 1 problem of many plantations. Ditches have to be kept cleared. The material removed from the ditches tends to build banks or small leeves beside the ditches, hindering drainage from the fields and causing trouble in operating machinery.

Turtlebacking is also called cut crowning by agricultural engineers. As the popular name suggests, it consists in grading all the land between two lateral ditches so that there is a low crown half way between with water free to flow to the ditches. Where ditch banks have been raised by repeated clearing of the waterways, this may call for moving considerable soil to the slightly raised center of the turtle back. Once the grading is completed most of the maintenance can be done by ploughing and dragging when fields are replanted. In the main, turtle backing really amounts to restoring to the sugarcane fields the fine soil that the heavy rains wash into the drainage ditches.

In the first field on which turtlebacking was tested on the St. Delphine Plantation, the improved drainage resulted in just about doubling the corn yield and an increase in the yield of cane equivalent to more than 900 pounds of sugar to the acre, or more than $50 an acre in the value of the crop.

SCS technicians are experimenting with various combinations of plows and graders to find the most economical way to turtleback the sugar fields that need this treatment.

R. H. O.

Synthetic Fibers Other Than Rayon Increasing Rapidly in Importance.—Synthetic fibers other than cellulose, in their infancy before World War II, are becoming increasingly important in the textile economy of the United States, according to a survey just made at the United States Department of Agriculture's Southern Regional Research Laboratory. The survey, which brings together the first figures to be made public on the consumption of these fibers, is part of a study of trends in the consumption of fibers in the United States, being made by Robert B. Evans and Barkley Meadows in connection with the research program on cotton utilization of the Bureau of Agricultural and Industrial Chemistry.

Until about 1935, rayon (synthetic fiber made of cellulose) was the only manufactured fiber in commercial production in this country. Its production, following one or two unsuccessful attempts, was started in the United States in 1911 and has increased so steadily and rapidly that as a textile fiber rayon now ranks second, quantitatively, only to cotton, with consumption increasing from 482 million pounds in 1940 to 875 million pounds in 1946.

Prior to 1940, the total consumption of all noncellulosic synthetic fibers (or synthetic fibers other than rayon) was so small in amount that it can be considered inconsequential. Thereafter, however, consumption increased rapidly, climbing steadily from 4.5 million pounds in 1940 to 53.3 million pounds in 1946. Although even the latter quantity is small as compared with the amounts of cotton, rayon, and wool consumed, it is already greatly in excess of the consumption of silk or flax.

Glass fiber is the oldest of the noncellulosic man-made fibers. Its production began in about 1936, but the output was quite limited during the first three or four years. Production of Vinyon, a fiber made of Vinylite synthetic resin, began about two years later, but is still relatively small. The commercial production of nylon yarns began in December 1939, following a short period of pilot-plant operations.

Manufacture of the casein fiber, Aralac, was initiated on a pilot-plant basis in 1939, with production in a commercial-size unit beginning in 1941. Saran, another fiber made from synthetic resin that was introduced commercially in 1940, might be termed a "borderline" case as a textile fiber since thus far it has been made commercially only in monofilament form; textile yarns are generally made up of many fibers or filaments. Since it frequently is woven and used for textile purposes, however, Saran has been included in the compilation given below. Certain other synthetic fibers such as soybean fiber and fiber made of acrylate resins, although in a pilot-plant stage of development, have not yet been produced on a true commercial scale and have not been included.

The accompanying table shows total consumption of synthetic fibers other than rayon in the United States for the years 1940–46, as indicated by reports from manufacturers. Separate figures for each specific fiber cannot be given since they would disclose the operations of individual firms.

Consumption of Synthetic Fibers Other Than Rayon [1] *in the U.S., 1940–46.*

Year	Quantity 1,000 pounds
1940	4,471
1941	11,663
1942	23,743
1943	37,200
1944	47,368
1945	49,292
1946	53,329

[1] Including domestic sales of nylon yarn and Aralac staple, estimated consumption of Saran as a textile fiber, production of Fiberglas continuous filament yarn and staple, and consumption of Vinyon in the United States.

Compiled from confidential reports of manufacturers except Fiberglas total for 1940, which was partially estimated.

R. H. O.

Plant Scientists Report on Chemical Quackgrass Killer.—Steady progress in the field of chemical weed eradication, according to plant scientists of the U. S. Department of Agriculture, promises ultimate control of many of the farmer's worst field pests. Latest advances have to do with the possibility of control of grassy weeds—such as quackgrass. The weed grasses are not controlled by the now well-known 2, 4-D.

Recently claims have been made for the effectiveness of IPC (isopropyl-N-phenyl carbamate) as a quackgrass killer. British scientists during the war found this substance, which is in the class of hormone-like chemicals or plant-growth regulators, checks or kills some kinds of cereals, members of the grass family, and that many broad-leaved plants are not injured by it. This led to the assumption that at least some grasses, including weed grasses, might be killed by applications of IPC, and now researchers at the Plant Industry Station, Beltsville, Md., announce favorable results from its use on mature quackgrass (with stolons or runners) and on quackgrass seedlings.

Limited experiments conducted at the Station since 1945 show that well established first-year quackgrass plants grown from seed and other quackgrass plants developed from well established stolons were killed by applying IPC to the surface of the soil. The experimenters, John W. Mitchell, P. C. Marth and L. W. Kephart, applied the chemical dry both outdoors and in the greenhouse, at varying rates, from 5 pounds to 60 pounds to the acre, using sand as a carrier. Even at the lowest rate of five pounds to the acre (in the greenhouse) the growth of shoots from stolons and of seedlings was completely checked. In 6 weeks after treatment outdoors at the rate of 10 pounds of IPC to the acre all growth was dead—not only the sprouts but the runners. In the greenhouse the stolon sprout growth was completely checked in 3 weeks and the stolons killed in 2 months with as little as 5 pounds to the acre.

Experience in the tests indicates, according to Mitchell, Marth, and Kephart, that the effect of the chemical on the plant is primarily from absorption through the roots. Consequently applications to the soil surface have been more effective than applications to the leaves. Experimentally, sand has proved a good material with which to mix the chemical as it readily sifts down to the soil surface.

The IPC (isopropyl-N-phenyl-carbamate) can be purchased as a fine powder from some chemical supply houses. The product may soon be found in packages under special trade names.

So far experience with this grass killer indicates the need for care as it will interfere with the growth of useful grasses for some time if the soil is dry. It is inactivated in moist soil, say the research men, at about the same rate as 2, 4-D, but so far little detailed knowledge has been built up on proper practice.

The experimenters warn that little is known so far about what effects may be produced by IPC on most broad-leaved plants. However, results at the Plant Industry Station have confirmed the English observations and that of others that some broad-leaved plants, including sugar beets and some weeds, have shown no signs of injury by it. They have observed no toxic effects to persons using IPC, but recommend reasonable care until more is known about effects on humans and animals.

R. H. O.